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Complex Analysis – Homework 1

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1. Exercise (4 points)

Sketch the following sets in the complex plane:

1.) $\mathcal{M}_1 := \{z \in \mathbb{C} \mid |3z - 1 + i| \leq 2\}$,

2.) $\mathcal{M}_2 := \{z \in \mathbb{C} \mid 0 \leq \mathrm{Im}(z) \leq 2\pi,\ |\mathrm{Re}(z)| < 1\}$,

3.) $\mathcal{M}_3 := \{z \in \mathbb{C} \mid \mathrm{Im}((1-i)z) = 0\}$,

4.) $\mathcal{M}_4 := \{z \in \mathbb{C} \mid |z - z_0| = |z - z_1|\}$, $z_0, z_1 \in \mathbb{C}$.

2. Exercise (4 points)

Investigate where the following functions are complex differentiable and determine their derivatives at these points:

1.) $f_1(z) := z\mathrm{Re}(z)$,

2.) $f_2(z) := z\overline{z}$,

3. Exercise (4 points)

Show the following statement: Let $\Omega \subseteq \mathbb{C}$ be a domain, let $f : \Omega \to \mathbb{C}$ be real differentiable, and let f fulfill the Cauchy–Riemann equations. Then f is complex differentiable (in Ω).

4. Exercise (4 points)

Let $\Omega \subseteq \mathbb{C}$ be open and $f : \Omega \to \mathbb{C}$ real differentiable. Show:

1.) f is holomorphic in Ω if and only if $f_{\overline{z}} \equiv 0$, and in this case, $\frac{\partial f}{\partial z} f(z_0)$ is the complex derivative of f for all $z_0 \in \Omega$.

2.) It holds that

$$\frac{\partial^2 f}{\partial z \partial \overline{z}} = \frac{1}{4}\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right).$$

Total: 16