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WiSe 21/22

Status: February 11, 2022

Differential Geometry I – Homework 12

Submission: February 18, 2022, 8 pm, to `Henriette.Lipschuetz@fu-berlin.de`

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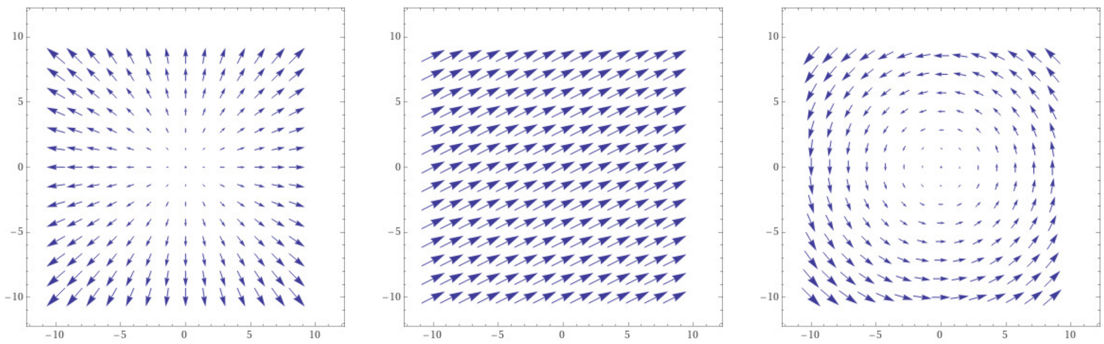
1. Exercise (8 points)

1.) Let $f : \mathbb{R}^3 \to \mathbb{R}$ a scalar field and $v : \mathbb{R}^3 \to \mathbb{R}^3$ be a vector field. Both are supposed to be twice continuously differentiable. Show:

a) $\operatorname{div}(\operatorname{curl}(v)) = 0$,

b) $\operatorname{curl}(\operatorname{grad}(f)) = 0$.

2.) Consider the following three representations of three vector fields in $\mathbb{R}^2$:



a) Find explicit representations v_i, $i \in \{1, 2, 3\}$, defined on the open square U approximating the three vector fields shown above.

b) Compute for the v_i, $i \in \{1, 2, 3\}$, derived above:

i.) $\operatorname{curl}(v_1) = 0$,

ii.) $\operatorname{div}(v_2) = \operatorname{curl}(v_2) = 0$,

iii.) $\operatorname{div}(v_3) = 0$.

Please, turn over.

2. Exercise (8 points)

Consider the given flat triangulation M_h shown below with vertices

$$P_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix},\ P_i = \begin{pmatrix} \cos\left(\frac{2\pi\cdot k}{6}\right) \\ \sin\left(\frac{2\pi\cdot k}{6}\right) \end{pmatrix},\ k \in \{0,\dots,5\}.$$

Let three vector fields $V^i \in \Lambda_h^1(M_h)$ on M_h, $i \in \{1,2,3\}$ be given by

* V^1 is given by $v_i = J\left(P_{i+1} - P_i\right)$ for $i \in \{1,\dots,6\}$ and $P_7 = P_1$,
* V^2 is given by $v_i = \left(\begin{smallmatrix} 2 \\ 1 \end{smallmatrix}\right)$ for $i \in \{1,\dots,6\}$,
* V^3 is given by $v_i = P_{i+1} - P_i$ for $i \in \{1,\dots,6\}$ and $P_7 = P_1$.

For these V^i, $i \in \{1,2,3\}$, do the following:

1.) Sketch V^1, V^2, and V^3.

2.) Determine for V^1, V^2, and V^3 both, curl and div.

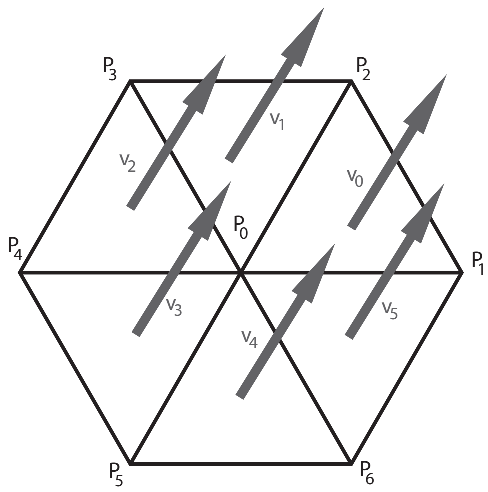


Figure 1: Not a scale model with an example vector field.

Total: 16