Freie Universität Berlin WiSe 21/22
Institut für Mathematik
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Differential Geometry I – Homework 11

Submission: February 10, 2022, 12:15 pm, to `Henriette.Lipschuetz@fu-berlin.de`

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1. Exercise (4 points)

Let M_h be a closed simplicial surface with a finite triangulation $\mathcal{T}$. For a vertex $v \in \mathcal{T}$ the *discrete graph Gauss curvature* K_h is defined by

$$K_h(v) := 6 - \deg(v),$$

where $\deg(v)$ denotes the degree of v in the 1-*skeleton graph* of $\mathcal{T}$, i.e. the number of adjacent edges.

a) Show that the following version of a discrete Gauss-Bonnet theorem holds:

$$\int_{M_h} K_h := \sum_{v \in \mathcal{T}} K_h(v) = 6\chi(M_h).$$

b) Show that the graph based Gauss Bonnet theorem stated above holds for the discrete Gauss Bonnet theorem stated in te lecture if all triangles are equilateral.

2. Exercise (4 points)

Consider the triangle Δ given by the three vertices $p_1 = \binom{-1}{1}$, $p_2 = \binom{-1}{-1}$, and $p_3 = \binom{2}{-1/2}$.

a) Determine the *hat functions* φ_i, $i \in \{1, 2, 3\}$, introduced in the lecture on Δ explicitly.

b) Illustrate your results.

c) Find the linear combination of a constant function on Δ.

d) Find the barycenter.

3. Exercise (8 points)

Let $\mathcal{M}$ be a geometric simplicial complex and define S_h as

$$S_h(f) := \{f : \mathcal{M} \to \mathbb{R} \mid f \text{ is (affine) linear on each } \sigma \in \mathcal{M} \text{ and continuous on } \mathcal{M}\}.$$

Show that S_h is real vector space (equipped with pointwise addition and scalar multiplication).

Total: 16