Freie Universität Berlin WiSe 21/22
Institut für Mathematik
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Differential Geometry I – Homework 10

Submission: February 3, 2022, 12:15 pm, to `Henriette.Lipschuetz@fu-berlin.de`

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1. Exercise (4 points)

Let $\Omega \subseteq \mathbb{R}^2$ be an open domain, $f : \Omega \to \mathbb{R}^3$ a parameterized surface, and $B \subset \Omega$ a compact subset. Further, let $N : \Omega \to \mathbb{S}^2$ denote the Gauß map. Assume N to be injective and DN to have full rank. Show

$$\int_{f(B)} |K| = \operatorname{area}(N(B))$$

where $\operatorname{area}(N(B))$ denotes the area hit by the Gauß map on $\mathbb{S}^2$.

2. Exercise (4 points)

Show: If all geodesics of a connected surface are planar, the surface is contained in the plane or the sphere.

3. Exercise (5 points)

1.) Show that every m-simplex has 2^{m+1} faces, i.e. subsimplices for $k \in \{0, \dots, m\}$.

2.) The Euler charcteristic can be translated to higher dimensions via defining

$$\chi(K) = \sum_{k \geq 0} (-1)^k f_k(K)$$

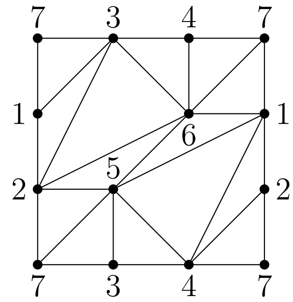
where f_k denotes the number of k-faces of a simplicial complex K. Let Δ^n denote the n-simplex. Show $\chi(\Delta^n) = 1$.

Please turn over.

4. Exercise (3 points)

The *Császár torus* is a two-dimensional simplicial complex consisting of 7 vertices, 21 edges, and 14 triangles[1].

This simplicial complex is shown in the following figure. Vertices labeled with the same index are identified and the edges are identified accordingly.



* Determine star([2]), star([3, 4]), and star([1, 5, 6]).
* Sketch your results.

Total: 16

[1] An embedding of the Császár torus is shown at `eg-models.de/models/Classical_Models/2001.02.069/`. The JVX-file can be opened in JavaView.