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# Differential Geometry I – Homework 09

Submission: January 27, 2022, 12:15 pm, to `Henriette.Lipschuetz@fu-berlin.de`

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## 1. Exercise (10 points)

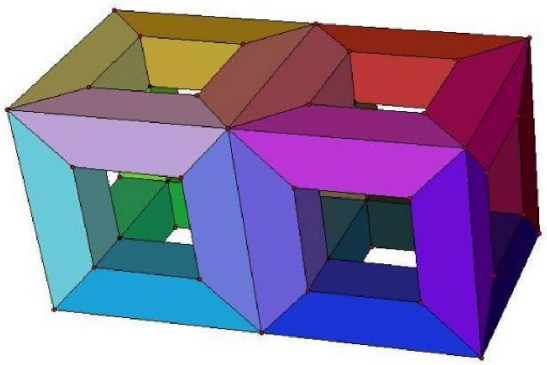
1.) Sketch[1] two nonisomorphic examples of a closed simplicial surface with 20 triangles and 12 vertices which are both not the icosahedron. Why are your examples nonisomorphic?

2.) How many edges does a simplicial double torus with 1200 vertices have got?

3.) Consider properties of quadrangulated surfaces.

a) Let $Q$ be a closed quadrangulated surfaces (such as the example shown in the next part of the exercise). Show that the Euler formula $\chi(Q) = |V| - |E| + |F|$ still holds.

b) Consider the following quadrangulated surface $T$:



Determine the number of vertices $v$, the number of edges $e$, the number of faces $f$ and the Euler characteristic $\chi(T)$. Further, determine its genus $g(T)$.

4.) Give three examples for simplicial surfaces (probably with boundary) having the same Euler characteristic but which are not simplicially isomorphic[2]. Justify your choice.

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[1] Think of an appropriate representation.
[2] I.e., they are not isomorphic as simplicial complexes.

**2. Exercise** (3 points)

Let $p$ be an inner point of a simplicial regular[3] surface $S$.

1.) Determine the number of triangles incident to $p$ such that the discrete Gauss curvature $K$ in $p$ is equal to $\frac{2\pi}{3}$, 0, or $-\frac{2\pi}{3}$ resp. Illustrate your results.

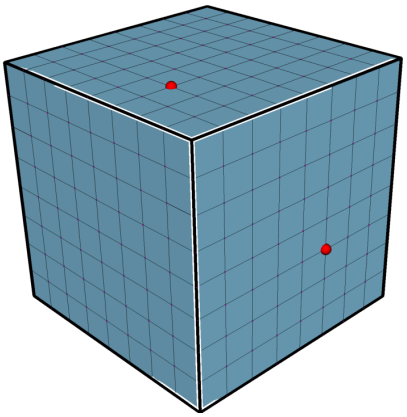
2.) Determine the number of triangles incident to $p$ such that $K(p) = 42\pi$ resp. $K(p) = -42\pi$.

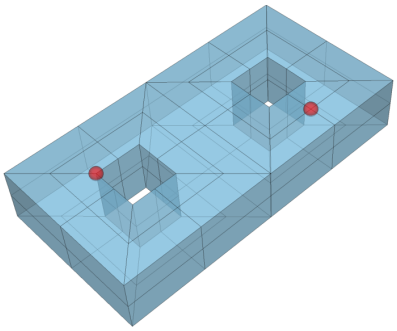
3.) Determine the discrete Gauss curvature for the surface depicted in Exercise 1, 3.), b).

**3. Exercise** (3 points)

1.) For the cube shown below (left image) find and sketch two straightest discrete geodesics connecting the two highlighted points, one of them being a shortest, the other one not being a shortest geodesic. Justify your solution.

2.) Consider the quadrangulated surface of genus 2 (right image). Find and sketch three straightest geodesics connecting the two highlighted points.





Total: 16

[3] All triangles are equilateral triangles.