Freie Universität Berlin
Institut für Mathematik
Prof. Dr. K. Polthier, H.-S. Lipschütz

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Differential Geometry I – Homework 08

Submission: January 20, 2022, 12:15 pm, to `Henriette.Lipschuetz@fu-berlin.de`

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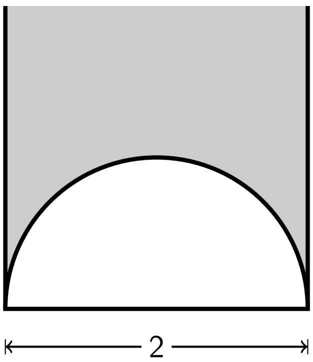
1. Exercise (3 points)

Consider the following geodesic triangle[1] in the Poincaré half plane model $\mathbb{H}$ whose vertices are lying all at infinity:



Determine its area by integration.

2. Exercise (8 points)

Consider the so-called *Cayley map*

$$f_C : \hat{\mathbb{C}} \to \hat{\mathbb{C}}, \quad z \mapsto \frac{z-i}{z+i}.$$

1.) Sketch the images under f_C of those lines which are parallel to the real or imaginary axis.

2.) Determine f_C^{-1}.

3.) Conclude that f_C is a bijection from the upper half-plane $\mathbb{H} := \{z \in \mathbb{C} \mid \operatorname{Im}(z) > 0\}$ to the unit disk $\mathbb{D} := \{w \in \mathbb{C} \mid |w| < 1\}$.[2]

4.) Determine the coordinates of an arbitrary point (x, y) in the half-plane model that is mapped to the disk model.

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[1] The triangle is given by the part colored in gray.

[2] This relates the Poincaré half-plane model to the Poincaré disk model.

3. Exercise (5 points)

1.) Construct a polyhedral torus using squares only. Illustrate your result.

2.) Count the vertices of positive, negative, and zero angle defect.

3.) Determine its Euler characteristic χ.

4.) Determine its total discrete Gaussian curvature.

Total: 16