Freie Universität Berlin WiSe 21/22
Institut für Mathematik
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Differential Geometry I – Homework 07

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1. Exercise (7 points)

Let $z_0 = 0$ and $F, G : \mathbb{C} \to \mathbb{C}$ be given by $F(z) = 1$ and $G(z) = z$.

a) Use Weierstraß parameterization for F and G as given above to derive a parameterization of the corresponding surface $f : \mathbb{R}^2 \to \mathbb{R}^3$, $(u, v) \mapsto (f_1(u, v), f_2(u, v), f_3(u, v))$.[1]

b) Show that all surfaces of the *associate family* $(e^{i\varphi}F, G)$, $\varphi \in [0, 2\pi)$, are intrinsically isometric. Plot the corresponding members of the associate family for $\varphi_1 = 0$, $\varphi_2 = \frac{\pi}{4}$, $\varphi_3 = \frac{\pi}{2}$, and $\varphi_4 = \pi$.

c) For $k \in \{1, 2, 3\}$, plot f on disk-shaped domains $D_k \subset \mathbb{R}^2$ such that f is

1.) embedded,

2.) nearly intersecting,

3.) a very large disk.

2. Exercise (4 points)

Show that the only surfaces of revolution (up to rigid motions and scaling) which are minimal are the plane and the catenoid.

3. Exercise (5 points)

a) Let $\Omega \subset \mathbb{R}^2$ be open and $f : \Omega \to \mathbb{R}$ be a harmonic function. Show: If f has a local maximum or minimum in Ω, then f is constant.

b) Conclude that a minimal surface with a local maximum or minimum in normal direction must be the plane.

Total: 16

[1] This surface is called *Enneper surface*.