Freie Universität Berlin WiSe 21/22
Institut für Mathematik
Prof. Dr. K. Polthier, H.-S. Lipschütz Status: December 7, 2021

Differential Geometry I – Homework 06

Submission: December 16, 2021, 12:15 pm, to `Henriette.Lipschuetz@fu-berlin.de`

---

1. Exercise (6 points)

The one parameter family of surfaces $f : [0, 2\pi] \times (-\infty, \infty) \times [0, \pi] \to \mathbb{R}^3$

$$(u, v, t) \mapsto \begin{pmatrix} \cos(t)\cos(u)\cosh(v) + \sin(t)\sin(u)\sinh(v) \\ -\cos(t)\sin(u)\cosh(v) + \sin(t)\cos(u)\sinh(v) \\ \cos(t)v + \sin(t)u \end{pmatrix}$$

describes a transformation of the *catenoid* $f(.,.,0)$ into the *helicoid* $f(.,.,\pi/2)$. Show that this transformation has the following properties:

1.) The surface normals remain unchanged, i.e. $\frac{\partial N}{\partial t} = 0$.

2.) All surfaces $f(.,.,t)$ are isometric, i.e. $\frac{\partial g}{\partial t} = 0$.

3.) The mean curvature vanishes for all u,v, and t.

2. Exercise (4 points)

Let f be a parameterized surface with unit normal N. Its *parallel surface at distance* $\epsilon > 0$ is defined by

$$f_\epsilon(u, v) := f(u, v) + \epsilon N(u, v).$$

Show that the area elements dA of f and dA_ϵ of f_ϵ are related via

$$dA_\epsilon = \left(1 - 2H\epsilon + K\epsilon^2\right) dA,$$

where H denotes the mean curvature and K the Gaussian curvature of f.

3. Exercise (6 points)

Consider the graph of $\tilde{f}(x, y) := x^3 - 3xy^2$, i.e. $f : (-1, 1)^2 \to \mathbb{R}^3,\ (x, y) \mapsto (x, y, \tilde{f}(x, y))$.

1.) Show that the graph of f is invariant under rotary reflections by $\frac{\pi}{3}$ with respect to the z-axis.

2.) Determine the principal curvatures at $x = y = 0$.

3.) Show that there are three distinct lines of curvature meeting at the origin by showing that the considered graph is mirror symmetrical to the (x, z)-plane.

Total: 16