

Differential Geometry I – Homework 05

Submission: December 2, 2021, 12:15 pm, to Henriette.Lipschuetz@fu-berlin.de

1. Exercise

(7 points)

The rotational symmetric catenoid can be parameterized by

$$f : [0, 2\pi] \times (-\infty, \infty) \rightarrow \mathbb{R}^3, (u, v) \mapsto \begin{pmatrix} \cosh(v) \cos(u) \\ -\cosh(v) \sin(u) \\ v \end{pmatrix}.$$

- 1.) Use some software (Geogebra, JavaView, etc.) to generate a plot of the catenoid.
- 2.) Verify whether the given parameterization is...
 - a) a conformal parameterization,
 - b) a curvature line parameterization,
 - c) providing geodesic parallel coordinates.

2. Exercise

(4 points)

Does there exist a surface $f_i : \Omega_i \rightarrow \mathbb{R}^3$, $i \in \{1, 2\}$, with the following first fundamental form g_i and shape operator S_i ? Justify your solution.

- 1.) $g_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $S_1 = \begin{pmatrix} 0 & 0 \\ 0 & u \end{pmatrix}$,
- 2.) $g_2 = \begin{pmatrix} 1 & 0 \\ 0 & \cos^2(u) \end{pmatrix}$ and $S_2 = \begin{pmatrix} 1 & 0 \\ 0 & \tan^2(u) \end{pmatrix}$.

3. Exercise

(5 points)

Find two examples of parameterized surfaces having the same Gaussian curvature, but which are not isometric to each other. Justify your results.

Total: 16