

## Differential Geometry I – Homework 03

Submission: November 18, 2021, 12:15 am, via e-mail to  
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### 1. Exercise

(7 points)

For  $t > 0$  consider the *tractrix*

$$(r, h)(t) := \left( \frac{1}{\cosh(t)}, t - \tanh(t) \right)$$

and the corresponding surface of revolution<sup>1</sup>

$$f : \Omega = \mathbb{R} \times [0, 2\pi), (t, \varphi) \mapsto (r(t) \cos(\varphi), r(t) \sin(\varphi), h(t)).$$

- 1.) Sketch the tractrix.
- 2.) Determine both principal curvatures  $\kappa_1$  and  $\kappa_2$ .
- 3.) Show that its Gaussian curvature is constant.
- 4.) Determine its surface area  $\int_{\Omega} \sqrt{\det(g)} dt d\varphi$ .

### 2. Exercise

(4 points)

- 1.) Show that a *surface of revolution* can always be parametrized such that  $E = 1$ ,  $F = 0$ , and  $G = g(t)$  where  $g$  is some function depending on  $t$  only.
- 2.) Show that a surface of revolution locally admits a *conformal parametrization*, i.e.  $E = G$ , and  $F = 0$ .

### 3. Exercise

(5 points)

Let  $f : \Omega \rightarrow \mathbb{R}^3$  be a parametrized surface with metric  $g$ , second fundamental form  $b$ , and shape operator  $S$ . At each  $p \in \Omega$ , the third fundamental form is a symmetric bilinear form  $h$  given by

$$h(v, w) := g(Sv, Sw) \text{ for all } v, w \in T_p\Omega.$$

Show the following equality

$$h(v, w) - 2Hb(v, w) + Kg(v, w) = 0,$$

where  $K$  denotes the Gaussian curvature and  $H$  denotes the mean curvature of  $f(\Omega)$ .

Total: 16

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<sup>1</sup>It is called *tractroid* or *pseudosphere*.