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SoSe 20

Status: July 8, 2020

Scientific Visualization – Homework 10

Submission: July, 16th, 2020, 10:15 am, via email

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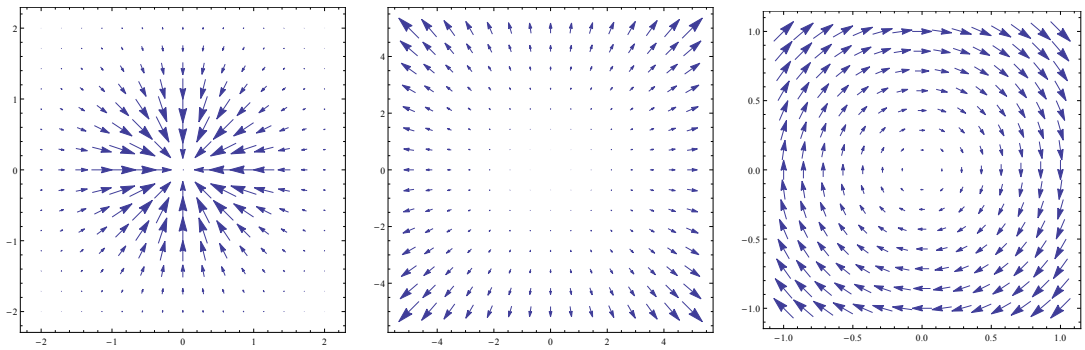
1. Exercise (8 points)

1.) Let $v : \mathbb{R}^3 \to \mathbb{R}^3$ be a vector field and $f : \mathbb{R}^3 \to \mathbb{R}$ a scalar field. Both are supposed to be twice continuously differentiable. Show:

a) $\operatorname{div}(\operatorname{curl}(v)) = 0$,

b) $\operatorname{curl}(\operatorname{grad}(f)) = 0$.

2.) Consider the following three representations of three vector fields in $\mathbb{R}^2$:



a) Find explicit representations defined on the open square U approximating the three vector fields shown above.

b) Try to find potential functions $f_i \in \mathcal{C}^1(U)$, $i = 1, 2, 3$, whose gradient fields $\operatorname{grad} f_i$ look as in the figures above, or explain if and why such a function cannot exist.

Please, turn over.

2. Exercise (8 points)

Consider the given flat triangulation M_h shown below with vertices

$$P_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix},\ P_i = \begin{pmatrix} \cos\left(\frac{2\pi\cdot k}{6}\right) \\ \sin\left(\frac{2\pi\cdot k}{6}\right) \end{pmatrix},\ k \in \{0,\ldots,5\}\ .$$

For the following two vector fields $V^i \in \Lambda_h^1(M_h)$ on M_h, determine the discrete divergence and rotation at the center vertex P_0.

1.) V^1 given by $v_0 = \ldots = v_5 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$;

2.) V^2 given by $v_0 = \begin{pmatrix} -1/2 \\ \sqrt{3}/2 \end{pmatrix}$, $v_1 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} -1/2 \\ -\sqrt{3}/2 \end{pmatrix}$, $v_3 = \begin{pmatrix} 1/2 \\ -\sqrt{3}/2 \end{pmatrix}$, $v_4 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $v_5 = \begin{pmatrix} 1/2 \\ \sqrt{3}/2 \end{pmatrix}$. Sketch V^2 as shown below.

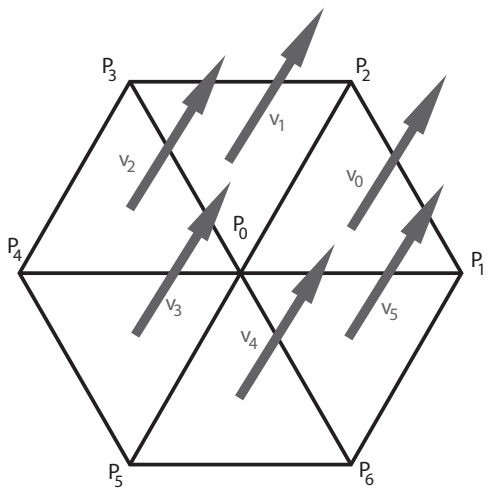


Figure 1: Not a scale model.

Total: 16