Freie Universität Berlin SoSe 20
Institut für Mathematik
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Scientific Visualization – Homework 9

Submission: July, 9th, 2020, 10:15 am, via email

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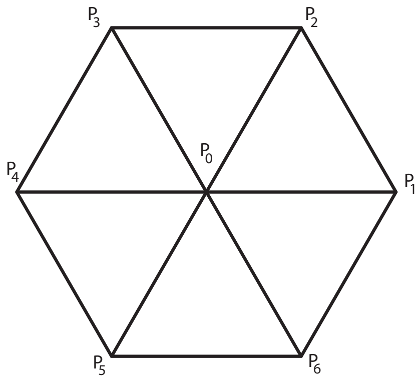
1. Exercise (8 points)

Consider the following triangulation with vertices

$$P_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, P_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, P_2 = \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}, P_3 = \begin{pmatrix} -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}, P_4 = \begin{pmatrix} -1 \\ 0 \end{pmatrix},$$
$$P_5 = \begin{pmatrix} -\frac{1}{2} \\ -\frac{\sqrt{3}}{2} \end{pmatrix} P_6 = \begin{pmatrix} \frac{1}{2} \\ -\frac{\sqrt{3}}{2} \end{pmatrix}$$

and let $u \in S_h$ be a continuous, piecewise (affine) linear function defined by $u_0 = 1$, $u_1 = 1$, $u_2 = 1$, $u_3 = \frac{1}{2}$, $u_4 = \frac{1}{3}$, $u_5 = 2$ and $u_6 = -3$, where $u_i := u(P_i)$.



1.) Determine the gradient field ∇u and sketch it.

2.) For the triangle $T_0 = [P_0, P_1, P_2]$, let $w \in S_h$ be a function with $w_1 = 1$, $w_2 = -1$ and gradient $\nabla w|_{T_0} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$. Determine w_0.

2. Exercise (8 points)

Let $\mathcal{M}$ be a simplicial complex and define S_h as

$$S_h(f) := \{ f : \mathcal{M} \to \mathbb{R} \mid f \text{ is (affine) linear on each } \sigma \in \mathcal{M} \} .$$

Show that S_h is real vector space (equipped with pointwise addition and scalar multiplication).

Total: 16