Scientific Visualization – Homework 4

Submission: May, 28th, 2020, 10:15 am, via email

---

1. Exercise (3 points)

Show that on $\mathbb{R}^3 \setminus \{0_{\mathbb{R}^3}\}$ an equivalence relation is given by

$$v \sim v' \Leftrightarrow \exists\, \lambda \in \mathbb{R} \setminus \{0\} : v = \lambda \cdot v'.$$

2. Exercise (8 points)

1.) Given the rotation R_φ around the angle $\varphi \in [0, 2\pi)$, the scaling S_λ by $\lambda \in \mathbb{R} \setminus \{0\}$, and the translation T_t by $t \in \mathbb{R}^2$ as (affine) maps $\mathbb{R}^2 \to \mathbb{R}^2$. Determine corresponding linear maps $\mathbb{R}^3 \to \mathbb{R}^3$ and R_φ^{-1}, S_λ^{-1}, T_t^{-1}, and $(R_\varphi T_t)^{-1}$ in terms of R_φ, T_t, and S_λ.

2.) Let $\mathcal{P} := \{P_1, P_2, P_3, P_4\} \subset \mathbb{R}^2$ be given by

$$P_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, P_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, P_3 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, P_4 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Determine an affine map $A : \mathbb{R}^2 \to \mathbb{R}^2$ such that $P_i \mapsto A(P_i) = Q_i$ for $i \in \{1, \ldots, 4\}$ and

$$Q_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, Q_2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, Q_3 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, Q_4 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

If possible determine A^{-1}.

3. Exercise (5 points)

Consider a simplicial surface defined by a set of points

$$P_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, P_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, P_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, P_4 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}.$$

with the triangles $\{P_1, P_3, P_2\}$, $\{P_1, P_4, P_3\}$, and $\{P_1, P_2, P_4\}$. Determine the error quadric Q_1 at point P_1 as defined in the algorithm by Garland and Heckbert.

Total: 16