Freie Universität Berlin SoSe 18
Institut für Mathematik
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Differential Geometry II – Homework 07

Submission: June 20th, 2018, 12:15 am

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1. Exercise (5 points)

Let (M, g) be a Riemannian manifold and let $\bar{M} \subset M$ be a 2-dimensional submanifold with normal field N. You may use without proof that

$$\bar{\nabla}_V W = \nabla_V W + b(V, W)N$$

where $b(V, W) := \langle \nabla_V N, W \rangle = -\langle \nabla_V W, N \rangle$.

1.) Show that $\langle \bar{R}(V, W)X, Y \rangle = \langle R(V, W)X, Y \rangle + b(V, Y)b(W, X) - b(V, X)b(W, Y)$.

2.) Let $\Pi_p \subset T_p\bar{M} \subset T_pM$ be a plane. Show that

$$\bar{K}(\Pi_p) = K(\Pi_p) + \det(b|_{\Pi_p}).$$

2. Exercise (6 points)

Let M be a manifold, $p \in M$ a point and denote by $\pi_1(M, p)$ the first fundamental group of M at p.

1.) Show that the group multiplication in $\pi_1(M, p)$ is associative.

2.) Give examples for M and p such that

a) $\pi_1(M, p)$ is Abelian,

b) $\pi_1(M, p)$ is not Abelian.

Justify your solutions.

3. Exercise (5 points)

By considering the covering $\mathbb{S}^2 \to \mathbb{R}P^2$, explain why $\pi_1(\mathbb{R}P^2)$ contains an equivalence class of a loop that is not contractible. Why does it become nullhomotopic if it is passed through twice? Illustrate your argument by providing a sequence af sketches of the homotopy.

Total: 16