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SoSe 18

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Differential Geometry II – Homework 03

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1. Exercise (5 points)

Let

$$\zeta :]-2,2[\times[0,2\pi] \to \mathbb{R}^3, (z,\varphi) \mapsto \begin{pmatrix} \cos(\varphi) \\ \sin(\varphi) \\ z \end{pmatrix}.$$

Let p be an arbitrarily chosen point in $\zeta(]-2,2[\times[0,2\pi]$. Determine the corresponding *exponential map* at p.

2. Exercise (5 points)

Show that the tangent space $T_p\mathrm{SO}(3)$ of the special orthogonal group

$$\mathrm{SO}(3) := \{A \in \mathbb{R}^{3,3} : A \cdot A^T = \mathrm{I}_3,\ \det(A) = 1\}$$

at the point $p = \mathrm{I}_3 \in \mathrm{SO}(3)$ can be identified with the set of skew symmetric 3×3 matrices.

3. Exercise (6 points)

Consider the space $\mathbb{R}^{n,n}$ of $n \times n$ matrices equipped with a norm satisfying $|AB| \leq |A||B|$. For $A \in \mathbb{R}^{n,n}$, the *matrix exponential* is defined as

$$\exp(A) := \sum_{k=0}^{\infty} \frac{1}{k!} A^k.$$

1.) Show that this series is convergent for any matrix A.

2.) Show that the matrix exponential maps skew symmetric matrices to orthogonal matrices.

3.) Show that the determinant of all matrices in the image of the exponential map is equal to +1.

Total: 16