Freie Universität Berlin WiSe 18/19
Institut für Mathematik
Prof. Dr. K. Polthier, H.-S. Lipschütz Status: January 7, 2019

Differential Geometry III – Homework 09

Submission: January 16, 2019, 12:15 am

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1. Exercise (4 points)

1.) Show: Let $G \subseteq \mathbb{R}^d$ be a region, and let $h : G \to \mathbb{R}$ harmonic. Then h achieves neither its maximum nor its minimum[1] (or h is constant).

2.) Show or disprove: The same statement holds in the discrete setting.

2. Exercise (4 points)

Let Ω be a unit square, and consider $f : \Omega \to \mathbb{R}$, $(x, y) \mapsto x + y$. Determine two different decompositions due to the Hodge decomposition. Determine the corresponding boundary conditions.

Total: 8

[1] *Hint:* You may use without proof: Let $G \subseteq \mathbb{R}^d$ a region, and $K \subseteq G$ compact. Then there exists $C \in [1, +\infty)$ depending solely on G and K such that $\frac{1}{C} \leqslant \frac{h(y)}{h(x)} \leqslant C$ for all $x, y \in K$ and h positive and harmonic.