

## Differential Geometry III – Homework 03

Submission: November 14, 2018, 12:15 am

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### 1. Exercise (4 points)

Let  $X, Y$  be two topological spaces and let  $\pi : X \rightarrow Y$  be a covering such that for  $x_0 \in X$  and  $y_0 \in Y$ ,  $\pi(x_0) = y_0$ . Show:

- 1.) If  $\gamma' : [0, 1] \rightarrow Y$  is a closed path such that  $\gamma'(0) = \gamma'(1) = y_0$  then there exists a unique lifting  $\gamma : [0, 1] \rightarrow X$  such that  $\gamma(0) = x_0$ .
- 2.) A homotopy  $h'$  of two closed paths  $\gamma'_1, \gamma'_2$  in  $Y$  which fixes the endpoints  $y_0$  lifts to a unique homotopy  $h$  of the lifted curves  $\gamma_1, \gamma_2$ .

### 2. Exercise (4 points)

Let  $X$  be a simply connected topological space and  $Y$  a topological space. Then the covering  $\pi : X \rightarrow Y$  induces a group isomorphism from the group of deck transformations into the fundamental group. Justify why  $X$  being simply connected is a necessary requirement.

Total: 8