Freie Universität Berlin WiSe 17/18
Institut für Mathematik
Prof. Dr. K. Polthier, H.-S. Lipschütz Status: January 29, 2018

Differential Geometry I – Homework 12

Submission: February, 7, 2018, 12:15 am

---

1. Exercise (4 points)
Construct an atlas consisting of two charts for the torus in $\mathbb{R}^3$ explicitly. Justify your solution.

2. Exercise (6 points)
Consider the 2-dimensional sphere

$$\mathbb{S}^2 := \{x \in \mathbb{R}^3 | x_1^2 + x_2^2 + x_3^2 = 1\}.$$

Show that $\mathbb{S}^2$ together with

$$M_1 := \mathbb{S}^2 \setminus \{(0,0,1)\} \quad M_2 := \mathbb{S}^2 \setminus \{(0,0,-1)\}$$

and $\varphi_i : M_i \to \mathbb{R}^2$, $i \in \{1,2\}$, where

$$\varphi_1(x) = \frac{1}{1-x_3}(x_1, x_2), \quad \varphi_2(x) = \frac{1}{1+x_3}(x_1, x_2),$$

is a differentiable manifold.

3. Exercise (6 points)
Construct an atlas for the projective plane $\mathbb{R}P^2$ explicitly. Justify your solution. Further, give an example of curve that is not continuously contractible. Again, justify your solution.

Total: 16