

Differential Geometry I – Homework 07

Submission: December 18, 2017, 12:15 am

1. Exercise

(8 points)

Let

$$f : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}^3, (u, v) \mapsto \left(u - \frac{u^3}{3} + uv^2, v - \frac{v^3}{3} + vu^2, u^2 - v^2\right).$$

Then f parametrizes the *Enneper surface*.

- 1.) Show that for f a *Weierstraß parametrization* is given by $F(z) = 1$ and $G(z) = z$.
- 2.) Show that all surfaces of the associate family $(e^{i\varphi}F, G)$, $\varphi \in [0, 2\pi)$, are intrinsically isometric and congruent in \mathbb{R}^3 .

2. Exercise

(4 points)

- 1.) Determine the Gauß curvature of the Lobachevski plane.
- 2.) Show that all geodesics in the Lobachevski plane have got infinite length.

3. Exercise

(4 points)

Show: if all geodesics of a connected surface are planar curves then the surface is contained in a plane or a sphere.

Total: 16