Freie Universität Berlin WiSe 17/18
Institut für Mathematik
Prof. Dr. K. Polthier, H.-S. Lipschütz Status: November 20, 2017

Differential Geometry I – Homework 05

Submission: November 27, 2017, 12:15 am

---

1. Exercise (6 points)

Let $f : \Omega \to \mathbb{R}^2$ be a parametrized surface and $c = f \circ \gamma : I \to f(\Omega)$ be a curve parametrized by arc length. Let (T, B, N) be the corresponding *Darboux frame*.

1.) Show that the normal curvature κ_n satisfies $\kappa_n = g(S\gamma', \gamma')$, and the geodesic curvature κ_g satisfies $\kappa_g = g(S\gamma', J\gamma')$. In the case that c is a Frenet curve its curvature satisfies $\kappa^2 = \kappa_g^2 + \kappa_n^2$.

2.) Show that c is a *line of curvature* (i.e. $c'(s)$ is a principal curvature direction for all s) if and only if the geodesic torsion vanishes, i.e. $\tau_g(s) = 0$ for all s.

3.) Show that if c is a Frenet curve and an *asymptotic line* (i.e. $\kappa_n(s) = 0$ for all s) then the geodesic torsion is equal to the torsion of the Frenet frame (i.e. $\tau_g(s) = \tau(s)$).

2. Exercise (4 points)

Does there exist a surface $f_i : \Omega \to \mathbb{R}^3$, $i \in \{1, 2\}$ with the following fundamental forms g and b?

1.) $g_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $b_1 = \begin{pmatrix} 0 & 0 \\ 0 & u \end{pmatrix}$,

2.) $g_2 = \begin{pmatrix} 1 & 0 \\ 0 & \cos^2(u) \end{pmatrix}$ and $b_2 = \begin{pmatrix} 1 & 0 \\ 0 & \sin^2(u) \end{pmatrix}$.

3. Exercise (2 points)

Determine the Christoffel symbols for the following surfaces:

1.) $f(u, v) = r(\cos(u)\cos(v), \sin(u)\cos(v), \sin(v))$, $r \in \mathbb{R}_+$,

2.) $f(u, v) = ((R + rcos(v))\cos(u), (R + r\cos(v))\sin(u), r\sin(v))$, $0 < r < R$.

4. Exercise (4 points)

Show: a regular closed curve $\gamma : [0, 2\pi] \to \mathbb{R}^2$ is a circle if and only if for all variations on closed curves with $\dot{L} = 0$ its area A fulfills $\dot{A} = 0$.

Total: 16