Freie Universität Berlin WiSe 17/18
Institut für Mathematik
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Differential Geometry I – Homework 03

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1. Exercise (6 points)
Let $\gamma : I \to \mathbb{R}^2$ be a planar Frenet curve with Frenet frame $\{T(t), N(t)\}$. Its *evolute* η is defined by

$$\eta(t) := \gamma(t) + \frac{1}{\kappa(t)} N(t).$$

1.) Compute the evolute of an ellipse: $\gamma_1 : [0, 2\pi],\ t \mapsto (a\cos(t), b\sin(t))$ where $a, b \in \mathbb{R}$.
2.) Compute and sketch the evolute of the cycloid:

$$\gamma_2 : (0, 2\pi) \to \mathbb{R}^2,\ t \mapsto (t - \sin(t), 1 - \cos(t)).$$

Interpret your results.

2. Exercise (5 points)
Let $\gamma : I \to \mathbb{R}^3$ be a Frenet curve with Frenet frame $\{T(s), N(s), B(s)\}$. For a smooth function φ the one parameter family of rotations

$$\begin{pmatrix} \tilde{N}(s) \\ \tilde{B}(s) \end{pmatrix} = \begin{pmatrix} \cos(\varphi(s)) & -\sin(\varphi(s)) \\ \sin(\varphi(s)) & \cos(\varphi(s)) \end{pmatrix} \begin{pmatrix} N(s) \\ B(s) \end{pmatrix}$$

generates a new orthonormal frame $\{T(s), \tilde{N}(s), \tilde{B}(s)\}$.
1.) Compute the torsion $\tilde{\tau}(s) = \langle \tilde{N}'(s), \tilde{B}(s) \rangle$ of the new frame $\{T(s), \tilde{N}(s), \tilde{B}(s)\}$.
2.) Determine some function φ such that the frame $\{T(s), \tilde{N}(s), \tilde{B}(s)\}$ is torsion-free, i.e. $\tilde{\tau}(s) = 0$ for all $s \in I$.
3.) Find a parallel frame for the helix parametrized by arc length (see Homework 02).

3. Exercise (5 points)
1.) For $a, b, c \in \mathbb{R}$ compute the first fundamental form of the following surfaces of rotation:
a) the ellipsoid:

$$f_1 : [0, 2\pi) \times [0, 2\pi) \to \mathbb{R}^3,\ (u, v) \mapsto (a\sin(u)\cos(v), b\sin(u)\sin(v), c\cos(u)).$$

b) the catenoid:

$$f_2 : \mathbb{R} \times [0, 2\pi], \ (u, v) \mapsto (a \sinh(u) \cos(v), b \sinh(u) \sin(v), c \cosh(u)).$$

2.) For $a \in \mathbb{R}_+$ compute the surface area of the following parametrized surface:

$$f_3 : (1, 2) \times [0, 2\pi), \ (u, v) \mapsto (u \cos(v), u \sin(v), av).$$

How is the surface called?

3.) (3 additional points) Plot[1] the surfaces corresponding to f_1, f_2 and f_3.

Total: 16

[1]Submit some printings.