Prof. Dr. Konrad Polthier
Dipl.-Math. Konstantin Poelke
Dipl.-Math. Thomas Gust

Differential Geometry I
Winter Semester 2014/15
Freie Universität Berlin

Exercise Sheet 10

Online: 17.12.2014

Due: 07.01.2015, 4:00pm

Exercise 10.1 (Vocab, 0 points)**.** Make sure you know for each item in the following list the definition and related theorems as well as the geometric intuition behind.

Vocabulary: *Arc length parametrization, Frenet equations, existence of curves with prescribed curvature and torsion, tangent space, first fundamental form, locally isometric, conformal parametrization, surface of revolution, Gauss map vs. normal vector field, second fundamental form, shape operator, principal curvature, Gauss curvature (smooth and discrete), mean curvature, curves on surfaces, Darboux frame, normal curvature, Meusnier's theorem, geodesic curvature, lines of curvature, asymptotic lines, normal variation, harmonic function, holomorphic function, Weierstrass representation, minimal surface, normal variation, geodesic, locally shortest,* Γ_{ij}^k*, integrability, Gauss equation, Mainardi-Codazzi, existence of surfaces with prescribed fundamental forms, Theorema Egregrium, Fermi coordinates, surfaces of constant Gauss curvature, Minding's theorem, Hilbert's Theorem, exponential map, hyperbolic halfplane model, Poincare disc, covariant derivative, covector field*

Exercise 10.2 (Geometry of Surfaces, 4 Points)**.** Imagine you talk to a fellow math student who has not attended a differential geometry course yet. Give a short (two sentences at most per item, no pathological endless sentences!) but precise answer for the following questions your fellow student might ask after you have told him a bit about differential geometry!

1. What is the difference between I and II?
2. What is the difference between II and L?
3. L is a self-adjoint operator, so why is its representing matrix l not symmetric?
4. What is an example for a geodesic that is not the globally shortest connection between two points?
5. What is an example for a parametrization whose parameter lines are not lines of curvature?
6. Is a line of curvature always an asymptotic line?
7. Is there a surface of revolution without any elliptic points?
8. What does it mean when you say that "Gaussian curvature is an intrinsic quantity"?

Exercise 10.3 (New Year, 0 Points)**.** Enjoy your Christmas holidays and have a good start into the new year!

Figure 1: "Circle Limit IV" - Illustration by M.C. Escher of the Poincaré disc model. Picture taken from http://www.mcescher.com/gallery/mathematical/circle-limit-iv/