Mathematics is everywhere

Eduardo Colli*, Fidel R. Nemenzo, Konrad Polthier, Christiane Rousseau†

Abstract. "Mathematics is everywhere" is the title for a panel at ICM 2014. The four panelists discuss what can be put under this title, what are the messages that can be passed to the public, and how to pass these messages. To most mathematicians, it seems obvious that mathematics is everywhere, and a living discipline within science and technology. Yet, how many of them are able to convey the message? And, when most people look around, they do not see mathematics, they do not know about the mathematics underlying the technology, they know very little about the role of mathematics in the scientific venture. Can we help building a powerful message? Can we unite forces for better passing it?

Mathematics Subject Classification (2010). Primary 00A05; Secondary 00B10.

Keywords. Popularization of mathematics, Mathematics a living discipline, Mathematics is everywhere.

1. Introduction

"Mathematics is everywhere."

Suppose you are a mathematician and you are put in front of this statement. Are you convinced? If you are in front of your classroom, are you able to explain the statement? And if you are in front of the public, or of a journalist, what examples will you choose to illustrate the statement? We, the panelists, have the impression that many of our mathematician colleagues are convinced. Yet, many of us lack good examples to pass the message. Indeed, the message should first please us before we decide to transmit it.

Let us now go to the schools. The teachers all know in principle that mathematical education is important but how can they answer the question "What is mathematics useful for, nowadays that calculators can do the computations, for us and that software solves the problems we used to learn to solve by hand?" How many teachers can take you to a tour of the city and show you the maths in all modern gadgets that you use, from a parabolic antenna, to a GPS, to the architecture of a building and the synchronization of trafic lights.

*Grant #2014/08628-1, São Paulo Research Foundation (FAPESP).
†This work was supported by NSERC in Canada.

If we now go to the public and claim that mathematics is everywhere, we could expect many skeptical faces...

Mathematics and its creative role in science, technology and society deserves to be better known. On the mathematical community side, this could result in more support of the society to mathematics, more interest of the kids in schools for their mathematics courses, and more interesting jobs for mathematically trained graduates. On the other side, everyone benefits from the contribution of mathematics to a better organization of our society, including public health system, management of resources, organization of transports. And mathematical breakthroughs in technology contribute to the creation of high technology companies.

This brings us to the gaps within our communities: gaps between mathematicians and other scientists, between pure and applied mathematicians, between researchers in mathematical sciences and mathematics educators. Perhaps the deeper gap lies between what mathematics is and how the public, including politicians and policy makers, sees it.

A challenge for the mathematical community is to convey the beauty and value of mathematics, that "mathematics is everywhere". Technology — used by people everyday — is always an opportunity to explain to the public the power of mathematics. More difficult is to convey the idea that abstract ideas too are beautiful and important and that a brilliant idea can make a breakthrough. At the most basic level, there is the need to restore everyone's (especially young people's) sense of awe and wonder.

Mathematicians should also be at the forefront of efforts to communicate mathematics, and bridge the gaps within the math community and society. International collaboration can make a significant difference. Preparing the right material for communicating mathematics requires energy and is time consuming, especially when it is hands-on. Also, anyone of us is always limited by his(her) own taste of mathematics. Putting the material on line allows sharing resources and using material on many different topics. Translating existing material can enrich significantly the material accessible in a given country. But it does not suffice that material be on line for it to be used...

Each of the four panelists will now present his(her) personal message. The paper will end with a conclusion relating the contribution to the theme that occurred during the panel itself.

2. Mathematics is everywhere

by Christiane Rousseau
Université de Montréal

This title is an extraordinary slogan. If you are not convinced, let us give a few examples.

Mathematics is everywhere in technology. Without mathematics, there would be no CT Scan. Indeed, a CT Scan only gives a series of numbers, namely the quantity of energy absorbed along the different rays through the body inside a plane, i.e. the Radon transform of the image, and the inverse Radon transform allows recovering a 2D image from these scattering data. Now, open your computer: Google's algorithm, which is so efficient, relies on the stationary distribution of a Markov chain: a clever idea created an empire. The small files for the images that you see on the Web have been compressed using Fourier transform in JPEG format, or wavelets in JPEG2000. Sensitive data is encrypted using number theory or algebraic geometry. And your computer is built with transistors that are sophisticated switches: building software ultimately comes to decomposing any operation into parallel sequences of elementary operations on 0 and 1. More examples in [5].

Mathematics is everywhere in science. Already for more than two thousand years, mathematics has evolved closely with physics, finding in physics a source of problems and providing solutions to physical problems. More recently, mathematics increased relationships with other sciences, and especially biology. The spectrum of applications in biology is immense, from the functioning of the cells or groups of cells and organs including the brain, to the functioning of the full body, with all the potential medical applications. Other types of applications include interactions of living populations, spreading and control of infectious diseases, ecology and ecosystems, and how biodiversity is organized on the planet.

The international year *Mathematics of Planet Earth 2013* (MPE2013) had a very important outreach component with the goal of showing the many applications of mathematics to, on one hand, discovering, understanding and managing our planet and, on the other hand, helping facing the planetary challenges of climate change and sustainability. The unprecedented collaboration around this international year is certainly explained by the increasing awareness among the public and the scientists that the planet is in real trouble, and that mathematics has a role to play in this issues. The theme is much wider than climate change and sustainability. Putting mathematical "glasses" we can discover the interior of the Earth by analyzing seismic waves generated by large earthquakes. Studying the planetary motions inside the solar system allows explaining the past climates of the Earth, and also the chaotic behavior of the inner planets, from which we cannot exclude, either a collision between two planets or expelling one planet from the solar system. The MPE2013 website (www.mpe2013.org) provides resources for enriching the curriculum and for outreach activities. This makes it easy for any teacher or professor to address such themes in secondary school, or even in undergraduate education. It is remarkable that MPE2013 occurred with almost no budget. This highlights how collaboration can significantly increase the impact of our outreach activities.

As mathematicians we work too much in isolation, and we should join forces with scientists to pass the message. I went in December 2013 to the Fall meeting of the American Geophysical Union and attended an education session. I was very impressed by the nice material that was presented, including numerical simulations: simulation of the formation of the Gran Canyon, simulation of a climate model in which the user could change the parameters, etc. Such material could easily enrich the curriculum in several of mathematics courses at the undergraduate level.

Having led MPE2013 since its inception in 2009, I was surprised by the number of mathematicians and teachers of mathematics who looked first excited by the theme and then disoriented if they had to produce examples. I would start listing a few applications simple to explain: how to calculate the length of the day depending on the season and the latitude, how a sundial works, provided that you correct time through the equation of time, how to draw a map of the Earth, how the GPS works, how to model the spread of an epidemic, etc. After a while, my vis-a-vis could continue the game and provide new examples I had not thought of. This means that even the convinced people need help to find good answers to questions like "What is mathematics useful for?", and "Has everything been found in mathematics?" Yet, these very important questions deserve significant answers. An answer could start with "Mathematics is everywhere.", then list a few applications where mathematics are hidden, before you continue with explaining the mathematics of your favorite application.

An example. I am a passionate of popularization of mathematics. I like to discuss examples and present strong scientific messages out of them. I like powerful ideas which are unifying in science. One of them comes from Turing's seminal paper "The chemical basis of morphogenesis"[6]. It is the idea that the loss of stability of an equilibrium through diffusion creates patterns. Take a flat stretch of dry sand and let the wind blow: there will always be a small irregularity that will stop some grains of sand, starting the beginning of a dune. Since the wind blows regularly, sand deserts are never flat, but rather covered with dunes. The same occurs with waves on the lakes and oceans, and with snow sastrugies in Antarctica. Turing idea's was that morphogenesis in biology has a chemical origin, with reaction diffusion phenomena involving several chemical reactants. Initially, the embryo has spherical symmetry, and the loss of stability of this equilibrium through diffusion leads to the formation of limbs, nose, ears, etc. The same model has been introduced in many areas from phyllotaxy when leaves appear along the stalk to the growth of plants modeled by L-systems ([1]). The L-systems have been introduced by the biologist, Aristid Lindenmayer. They model the growth of complex plants by iteration of a small number of operations, similar to iterations of a few instructions of a cellular automata (see Figure 1).

Reaction-diffusion models have also been proposed for modeling the fractal patterns appearing on some seashells ([2]), and for the patterns of animal coatings (see for instance Murray). A discrete version of the reaction-diffusion model can be given for the pattern of the Cymbiola Innexa REEVE (see Figure 2), which resembles a lot the Sierpinski carpet. The pattern is generated one line at a time, similar to the ridges of a real shell. There are two reactants: the activator (A) is colored and given the value 1, while the inhibitor (I) is white and given the value 0. If one divides the image into pixels, then the pattern is formed iteratively row by row. The color of a pixel is the sum modulo 2 of the two pixels that touch him

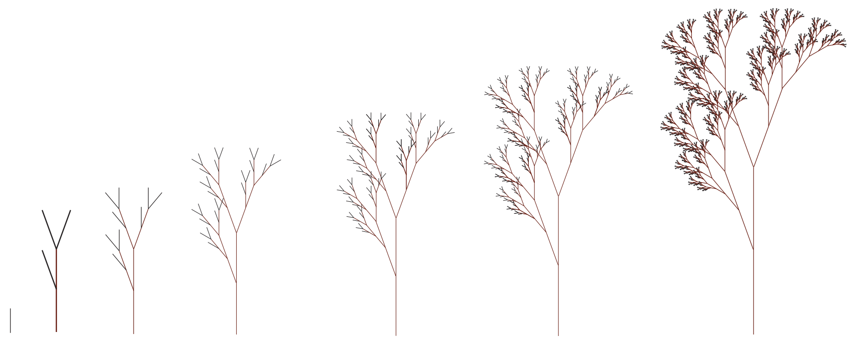


Figure 1. An example of L-system with two rules. On the figure, appears the initial condition, a segment of type F, and the first six iterations. The symbol S corresponds to a stalk piece and the symbol F to a terminal branch. The first rule corresponds to replacing a terminal branch by a stalk of length 2 and three terminal branches. It is given by: $F \mapsto S+[F]-S+[S]--[S]$. The second rule doubles the length of the stalk: $S \mapsto SS$. $\pm$ corresponds to a rotation of a given angle to the left or to the right, and the notation [branch] means that we must come back to the beginning of the branch before starting the next instruction.

by the corner on the preceding row.

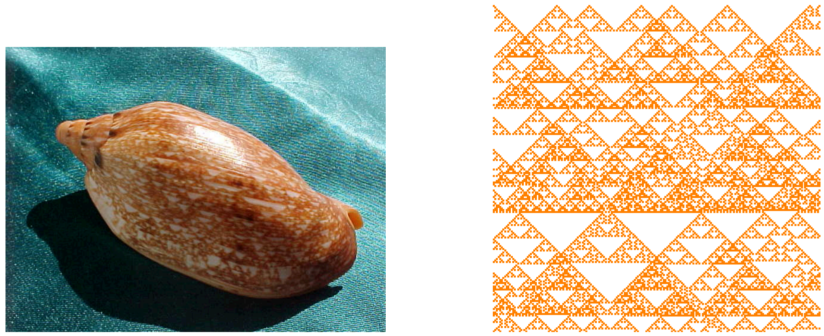
The reaction-diffusion model used to describe the patterns of animal coatings is especially interesting. The same model allows four different types of patterns (see Figure 3):

- spots;
- labyrinths;
- gaps;
- stripes.

For animal coatings, the patterns which appear depend only on the size and shape of the surface at the time of the pattern formation. In particular, stripes usually occur on thin tubular regions. It is especially striking that these four types of patterns are exactly the ones observed in vegetation patterns. Vegetation patterns occur when there is not enough water for full vegetation cover. Above a certain threshold of moisture, vegetation can survive. Spots are observed when the moisture is minimum, then labyrinths, then gaps. Stripes occur on slopes.

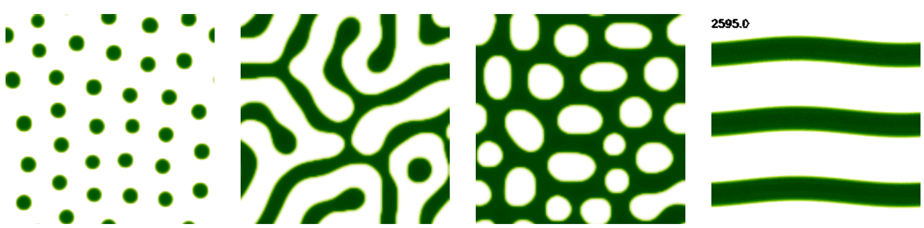
In chemistry, the most famous example is the Belousov-Zhabotinsky reaction-diffusion, an oscillating reaction creating regular patterns before moving to a chaotic behavior.

More recently, the model has also been used in the context of ecological invasions, spread of epidemics, tumor growth and wound healing. James Murray has been particularly active in all these applications.



(a) A Cymbiola Innexa REEVE (b) A pattern generated by the model

Figure 2. A Cymbiola Innexa REEVE (Photo credit: Ian Holden, Schooner Specimen Shells), and a pattern generated by computer, using the discrete reaction-diffusion model described.



(a) spots (b) labyrinths (c) gaps (d) stripes

Figure 3. The four types of patterns (images provided by A. Provenzale from papers [3] and [4]). The stripes are perpendicular to the slope.

3. Using math-glasses in Seoul: a walk in the city with eyes and mind tuned in on mathematics

by Eduardo Colli
Universidade de São Paulo

The slogan "math is everywhere" is commonly used in math popularization and is generally accepted and understood by those who teach mathematics. But when one is faced with the task of convincing someone that this assertion is true some philosophical issues arise: what exactly is meant by "everywhere"? What does it mean "is"?

I think that it is more or less a common sense that "everywhere" in real life is something near the concept of a dense set in mathematics. We cannot escape, in our urban life, of seeing matters which are closed interrelated with mathematical concepts. But on the other hand it would be rather pretentious to claim that every aspect of life or of human life is related to mathematics: think about a conversation between two old friends, telling each other how about their lives are in that day. Or think in a film, or in a romance, or in a play. Can we say there is math there? I would answer "no, at least based on what we know in the present days".

Moreover, what we mean by saying "there is math here or there"? In fact, there are some different ways of relate mathematics to what we see in real life. We give three.

1. One is by means of an application, understood as some technique which is developed by human beings in order to solve some problem. The GPS technology is an often cited example: it is based on the mathematical assertion that in three-dimensional Euclidean space if we know our distance to four points (the satellites), and these points are in generic position (in this case, there is no three of them lying in the same line and the four do not lie in the same plane) then our position is determined. In fact, three satellites suffice if the second solution can be discarded by other means. The implementation of this principle also uses more advanced mathematics, since relativity theory must be taken into account for precise measurement of these distances.

2. Other is by means of relating a phenomenon to a mathematical model, like a body in free fall. Nature is full of examples of patterns arising from simpler rules that generate them. Even if this hypothesis is controversial, I assume that mathematics *is* in these patterns.

3. A third is when some human being gets inspiration in mathematical concepts to bring some kind of delight to real life. This is common in architecture, design and art – see for example the Brazilia Cathedral, which is a hyperboloid of revolution, or the catenaries present in Gaudí's work, but it may also be the case in the creation of games – like Hex – and puzzles – like Rubik's cube. In these cases math appears purposely but with no intention of solving a problem.

These three categories are not completely disjoint. For example, an object of design may seem to be inspired in mathematics, but the designer itself created it without being aware of its mathematical relations. So the mathematics of the object appear as a model of the object, as it was created by nature. Other example is when a good mathematical model of a phenomenon can prove itself useful to applications only much later than its discovery – for example Newton laws of mechanics and gravity to explain the movement of planets and much later the launch of satellites and spaceships. Here the mathematics that we attach to the phenomenon brings the seed of a technical application.

At the end, mathematics is both language and science, it is both tool and inspiration, it is application and abstraction. Is is inherent to the human being and that is why it is everywhere.

The walk. It is endowed with this way of seeing – that we call here "math-glasses", a term already used elsewhere – that we will take a quick walk through Seoul. The reader will easily recognize the three above mentioned ways of seeing "mathematics everywhere" and I invite him/her to wear math-glasses from now on in every situation possible.

I won't reproduce pictures of the visited places, but rather indicate in the footnotes the internet sources where they can be found. One may also use keyword searches using the names of places and buildings. It should be clear that this text was written several months before ICM, so I will be honest to say that in fact I did not do a true walk, but an 'internet walk' in Seoul.

Modern architecture. Our first stop is in front of the (new) Seoul City Hall[1]. This building was opened in 2012 and was conceived by Yoo Kerl. It seems that mathematics have inspired the artistic realization of Kerl's motivations – see the composition with triangles, the approximation of an oval surface by polygons and the curved surface in the facade, resembling a graph of a two variable function, $z = f(x, y)$, with z in the horizontal direction, perpendicular to the facade.

But apart from the direct mathematical inspiration of this particular building, it calls our attention to the incredibly powerful tool that mathematics, together with computers, brought to architects, designers, engineers and other with CAD – Computer Aided Design. These softwares are the uttermost modern application of geometry, particularly analytic geometry, linear algebra and curve/surface approximation and interpolation techniques. Nowadays it is impossible to walk in an urban area, indoors or outdoors, without seeing buildings and objects that were first drawn in some software like these.

East Asia ancient architecture. Our next stop is at the hall of Changdeokgung Palace, a typical example of East Asia ancient architecture[2]. I was astonished to find that mathematics is also used to master the conception of new buildings in

[1] I like the picture of Minseok Kims Blog, at *linkwind.blogspot.com.br/2012/11/new-city-hall-of-seoul.html.*

[2] There is a nice picture at *en.wikipedia.org/wiki/Changdeokgung.*

this style, also taking into account the differences in style of japanese, chinese and korean cultures. In [7] and references therein it is discussed the procedural modeling of this kind of construction through CAD softwares where the user simply chooses some parameters and the basic structure of walls and roofs is automatically drawn following a prescribed algorithm.

In this case, human beings are modeling the creation of other human beings and, as a result, establishing a path to preserving cultural heritage.

New branches of geometry thinking: packing. If we have children with us, why not spending some time in a candy store, with its plastic cylinders fulfilled with colored candies? Those who wear math-glasses easily recognize matters related to sphere packing. Although children are mostly interested on colors, tastes and textures, the seller could be particularly wondering whether one of his/her packing cylinders lets less or more empty space accordingly to the size of the balls inside.

Reasoning in simple term shows that we may not expect great differences as a function of the ball sizes, since locally the ratio between empty and filled space is the same, independently of the size, as long as the balls have the same size. Of course we have to neglect the effects of the wall, but they are small when the balls are much smaller than the cylinder radius.

The sphere packing problem in infinite three-dimensional Euclidean space has challenged mathematicians for three centuries with the so called Kepler conjecture, stating that $\frac{\pi}{\sqrt{18}}$ is the best density possible for equal spheres. A computer-assisted proof has been provided by Hale and Ferguson, and they are working on a proof that could be checked by an automatic proof checker program[3].

Parabolas or not? Night is falling and we go take a look at the Moonlight Rainbow Fountain, at Banpo Bridge[4]. We immediately identify the parabolic shapes of the jets. Are they really parabolic? If there was not air resistance, they would be.

But suddenly each jet goes out with different slope[5]. What would we see if we were placed at the beginning of the bridge, looking sideways at the jets? (see Figure 3). We assume that despite the different slopes, the water goes out always with the same velocity. The jets fulfill a portion of the sight plane, which is bounded by a curve: the envelope. What curve does give the envelope? The answer is surprising: it is also a parabola!

The introduction of air resistance complicates things a little bit and trajectories are no longer parabolic. If it was a thrown object, a good model for air resistance is a force proportional to the square of the absolute velocity and opposite to the velocity vector. Therefore the parametrization of the trajectory would be the solution of the second order ordinary differential equation $(\ddot{x}, \ddot{y}) = (0, -mg) - \alpha \sqrt{\dot{x}^2 + \dot{y}^2}\, (\dot{x}, \dot{y})$.

[3] See *en.wikipedia.org/wiki/Kepler_conjecture.*

[4] *en.wikipedia.org/wiki/Banpo_Bridge*

[5] In the link *spreadsomeawesome.com/2011/09/29/han-gang-river-cruise-in-south-korea/* one can see a picture where the jet slopes vary sinusoidally.

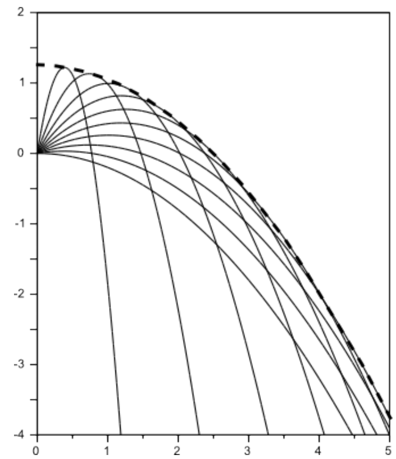


Figure 4. Parabolic fountain jets viewed from sideways. Each jet is launched at the same velocity V_0. The dotted line shows the envelope.

Notice also that each jet can have its own color, an effect that can be obtained injecting light aligned with the water outlets. Then light is imprisoned in the jet, which is a very interesting effect of total reflection, when the angle of the incident ray is so small that there is no refraction, a simple consequence of Snell's law. This principle is used in the construction of optical fibers.

Traffic jam. Now it is time to go back to the hotel, but traffic conditions are not favorable. Why are there traffic jams even in open roads without any traffic signals or car accidents?

The flow on a road can be modeled in many ways and one of them is supposing it is a continuous (see [9], for example). This continuous has three time-and-position-dependent relevant variables: flow $q = q(x,t)$, in cars per second, concentration $k = k(x,t)$, in cars per meter, and speed $v = v(x,t)$, in meters per second. These three variables are related by two simple equations, in such a way that it suffices to study only one of them. The space and time evolution of this remaining variable is then ruled by a partial differential equation, that can explain wave effects of the traffic. This equation is an example of conservative laws, which deserves a whole mathematical area on its own.

A group in Japan [11] shows experiment and analysis with real cars in a circular lane (a video recording can also be seen). From the video it is clear that, at least for a high concentration, constant concentration is an unstable state.

Subway queues. Giving up to get a taxi drive in the traffic jam, maybe the subway is a good alternative. But subways have queues in the platforms [8]. People arrive at the platform of train line obeying to some probabilistic distribution law, in general something like a Poisson distribution, that says the probability of k

people arrive between two train departures. On the other hand, there is a matrix assigning probabilities of getting out at a station given that the passenger got in at some other station.

With enough data the subway administration could then calculate the probability of having a queue exceeding the capacity of the platform, avoiding dangerous situations. Or they could carry on a careful study about the ideal time lapse between two trains.

A hands-on 3-sphere. I am tired and not willing to face these queues. Better spending time at a coffee shop, playing with a *souvenir* that I have found in some trinket shop, the magical folding cube (Figure 3)[6].

Figure 5. Magic folding cube.

But I am still wearing my math-glasses. I start wondering what if this object was a kind of spacecraft of 8 chambers (the smaller cubes), each face of the chamber with a door that opens only when there is another chamber at the other side of the wall. This situation is achieved by a suitable articulation of the spaceship that makes the walls touch. There is no door to the outside, only doors that connect one chamber to the other. An astronaut in this spacecraft, where does he/she live?

The answer is: he/she lives in a 3-sphere! The 3-sphere is the only compact orientable simply connected manifold of dimension three, but this we know only now, after G. Perelman has proven a conjecture of Poincaré that stood unanswered for more than one hundred years!

Conclusion. In this one day walk we saw geometry in its pure form or related to modern aspects like interpolation and packing; ordinary and partial differential equations; statistics and probability; and topology. Further exploration can go much further than I did in the above paragraphs. The examples could be not only directly used in classroom, but also be a source of inspiration in the task of seeking out day-by-day examples.

[6] Taken from *www.gyroscope.com*

4. Bridging gaps and communicating mathematics

by Fidel Nemenzo
University of the Philippines

In the late 50s, the British intellectual C.P. Snow, in his famous lecture 'The Two Cultures', lamented the fragmentation of the learning in the academe and the widening gap between the humanities/social sciences and the sciences. His lecture drew attention to the failure of specialists to communicate their ideas across the boundaries of their disciplines.

There are gaps too within our mathematical community, brought about by specialization and the different 'languages' we speak. There is the gap between 'pure' and 'applied' mathematics, between research mathematicians and mathematics educators.

But perhaps the deeper gap lies between those who do and teach mathematics on one hand, and those who use mathematics– everyone! This is a gap between what mathematics is and how the public sees it. The lack of understanding and appreciation of mathematics on the part of the public has grave implications on our students' education, our school systems and government policy. In some countries, funding support for mathematics and the basic sciences is dwindling, in favor of the applied disciplines which are seen to have more 'direct' benefits to industry and society. This underscores the need to communicate the beauty and power of mathematics to the broadest possible audience and convey the message that mathematics is part of almost every aspect of our lives.

Like ordinary language, mathematics allows one to represent and communicate ideas and meanings. It has been described as the language for the study of patterns about quantity, space and shape, and structure. Mathematics is abstract, but because of its precision, it is the language of science, helping us model and understand the natural and physical world and providing the ideas that power modern technology. In fact, it is also increasingly becoming part of the language for understanding and modeling social phenomena in a diverse range of disciplines such as economics and sociology.

Technology — ubiquitous and used by people everyday — gives us an opportunity to explain to the public the powers of mathematics.

Take for example, error-correction codes. Human beings are equipped with the ability to detect and correct errors, and thus we are able to read and correct the following corrupted message: "*you can raed a taotl mses wouthit a porbelm. This is bcuseae the human mnid deos not raed ervey lteter by istlef, but the word as a wlohe.*"Word processing programs too have error correction: MS Word can convert the mistyped word '*mathemaitsc*' to the correctly spelt 'mathematics', by identifying the corrupted word with the 'closest' item in its collection of legitimate words. This implies the use of some notion of 'distance' between words.

It is a safe assumption that errors occur whenever we transmit data or information across 'noisy' channels. Data can be text messages, digital images, sound or movie files, etc. Coding theory is the mathematics behind the packaging of information so that we are able to efficiently transmit the information, and detect and correct the errors. The idea is to construct abstract mathematical objects called 'codes' which are used to represent the data one wishes to transmit. Good codes are equipped with algebraic structure and some notion of 'distance' that allows efficient error detection and correction. The traditional codes are constructed as vector spaces over finite fields. But in the last two decades there has been growing interest in codes over finite rings. There is also a class of codes built from algebraic curves over finite fields.

Originating from the works of Shannon and Hamming during the mid-20th century, and clearly motivated by the requirements of engineering and information technology, research in coding theory draws ideas from many fields of mathematics, such as number theory, ring and field theory, linear algebra, combinatorics and geometry. Coding theory is the mathematics working behind the scene in many of the gadgets and machines we use everyday — such as mobile phones, CD and DVD players, etc.

Prime numbers — the building blocks of the integers — are both fascinating and mysterious. They have pleasing properties that are a source of delight for school children, amateur number enthusiasts and professional mathematicians. They also baffle, and give rise to many open problems in mathematics. One particular problem is computational: given an integer, decompose it as a product of prime factors (prime factorization). There is no known efficient algorithm for this, and the difficulty of prime factorization serves as the basis for the security of well-known public key cryptosystems, such as the RSA cryptosystem. Cryptosystems are methods of encrypting (and decrypting) information for secure transmission.

Another mathematical object used in cryptography is an elliptic curve, whose properties can be used to construct cryptosystems with higher security and shorter keys. Elliptic curves are smooth cubic curves whose points are endowed with some neat algebraic structure (over the rationals, a finitely generated abelian group) and arithmetic. Although they have been studied as asbstract objects for the past 150 years, elliptic curves have some surprising applications. Andrew Wiles' proof of Fermat's Last Theorem (1995) is based on the modularity of elliptic curves. About thirty years ago, elliptic curve cryptosystems were introduced as an alternative to the standard RSA-type methods ([12], [13]). A point on a curve (defined over finite fields) can easily be multiplied by an integer, but it is very difficult to compute the number, given the original point and the result. The security of elliptic curve cryptosystems is based on the difficulty of this computational problem.

Everyday, most people use and enjoy the benefits of technology such as their mobile phones, digital cameras and the internet, unaware these run on the power of mathematical ideas.

More than 50 years ago, the mathematician G.H. Hardy once rejoiced in the 'uselessness' of number theory, "whose very remoteness from ordinary human activities", he said, "should keep it gentle and clean." [14] Today, number theory — the study of numbers, such as prime numbers — is no longer seen as 'useless'. Like coding theory, it works behind the scenes in the internet, securing our email and financial transactions, authenticating sources of data, and ensuring the safe passage of information. This is the irreversible trend in mathematics — the divisions between the pure and the applied are breaking down. Abstract ideas in mathematics, developed for their own sake, are now finding new applications. And conversely, problems in physics, IT and engineering, the physical and biological sciences, are stimulating new research in mathematics.

While the main task of mathematicians is to do mathematics, they should share, with educators, the responsibility of promoting the right attitude towards mathematics, among students, the popular media, our governments, and the broader public. A well-informed public is necessary for a public culture that is supportive of mathematics. There are too many biases, fears and misconceptions out there about our discipline that should be dispelled in all possible arenas, including social media. Mathematicians should be at the forefront of communicating the delight, beauty and power of mathematics as both language and tool, and dispelling the stereotype of a mathematician as a performer of mental acrobatics, cut off from society. An educated public need not have a grasp of equations and formulas, but should understand the role of mathematics in shaping our world.

5. Conclusion.

The different contributions have highlighted the challenges facing our mathematical community. On the one hand, the suggested title of this panel, *"Mathematics is everywhere",* is a fascinating slogan that would deserve to be exploited more often, both in terms of strong messages in science and technology, but also as a game to be played when we look around us and dismantle what we see to discover the mathematics hidden in so many objects or phenomena around us. On the other hand, the image of our discipline still needs improvement, and our community is facing challenges mainly in terms of communication, collaboration between communities, and preparation of messages, resources and material to communicate. We should join forces to improve the situation, especially considering the fact that, with the web, it is easier than ever to share resources and spread the message to the largest public possible.

References

[1] Lindenmayer, A. and Prusinkiewicz, P., *The algorithmic beauty of plants*, Springer-Verlag, 1990 (pdf bversion can be downloaded for free at http://algorithmicbotany.org/papers/#abop).

[2] Meinhardt, H., *The algorithmic beauty of sea shells*, Springer-Verlag Berlin Heidelberg, 2009 (fourth edition).

[3] Gilad E., von Hardenberg J., Provenzale A., Shachak M., Meron E., Ecosystem engineers: from pattern formation to habitat creation, *Physical Review Letters*, **93** (2004), 098105 1–4.

[4] Gilad E., von Hardenberg J., Provenzale A., Shachak M., Meron E., A mathematical model of plants as ecosystem engineers, *J. of Theoretical Biology*, **244** (2007), 680–691.

[5] Rousseau C. and Saint-Aubin Y., *Mathematics and Technology.* Springer Undergraduate Series in Mathematics and Technology, Springer, NY, 2008.

[6] Turing A., The chemical basis of morphogenesis, *Philosophical Transactions of the Royal Society of London, Series B, Biological Sciences* **237** (1952), 37–72.

[7] Soon Tee Teoh, Generalized descriptions for the procedural modeling of ancient East Asian buildings, DOI:10.2312/COMPAESTH/COMPAESTH09/017-024. In proceedings of: Computational Aesthetics 2009: Eurographics Workshop on Computational Aesthetics, Victoria, British Columbia, Canada, 2009.

[8] Xin-yue Xu, Jun Liu, Hai-ying Li, Yan-fang Zhou, Probabilistic model for remain passenger queues at subway station platform, *J. Cent. South Univ.* **20** (2013), 837–844.

[9] John A. Adam, *X and the City. Modeling aspects of urban life.* Princeton and Oxford: Princeton University Press, 2012.

[10] Robert B. Banks, *Towing icebergs, falling dominoes, and other adventures in applied mathematics.* Princeton, New Jersey: Princeton University Press, 1998.

[11] Yuki Sugiyama, Minoru Fukui, Macoto Kikuchi, Katsuya Hasebe, Akihiro Nakayama, Katsuhiro Nishinari, Shin-ichi Tadaki and Satoshi Yukawa, Traffic jams without bottlenecks: experimental evidence for the physical mechanism of the formation of a jam, *New J. Phys.* **10** (2008), 033001. doi:10.1088/1367-2630/10/3/033001

[12] Neal Koblitz, Elliptic curve cryptosystems, *Mathematics of Computation* **48** (177) (1987), 203-209.

[13] Victor Miller, The uses of elliptic curves in cryptography, *CRYPTO* **85** (1985), 417-426.

[14] G.H. Hardy, *A Mathematician's Apology*, Cambridge University Press, 1940.

Eduardo Colli, Universidade de São Paulo
E-mail: colli@ime.usp.br

C. Rousseau, University of Montreal
E-mail: rousseac@dms.umontreal.ca

Konrad Polthier, Frei Universität Berlin
E-mail: Konrad.Polthier@fu-berlin.de

Fidel Nemenzo, University of the Philippines Diliman
E-mail: fidel@math.upd.edu.ph