Anisotropic Mesh Adaptation via Higher Dimensional Embeddings

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Anisotropic meshes are essential to deal with phenomena characterized by directional features. They offer a better compromise between computational effort and numerical approximation.

How to generate anisotropic meshes is a challenging problem. The most common strategy is to consider a prescribed (discrete) metric tensor field $\mathcal{M}$ defined over a mesh domain $\Omega \subset \mathbb{R}^d$. Each metric tensor $M \in \mathcal{M}$ is a $d \times d$ symmetric positive definite matrix, which contains the desired anisotropic information. Then, a *uniform mesh* with equal (geodesic) edge length with respect to $\mathcal{M}$ is generated, see, e.g., [4, 6, 8, 5].

In this talk, we exploit a new approach that is not based on a metric tensor field. The basic idea is to increasing the dimension of the space which contains the real mesh domain, such that the anisotropic features in $\mathbb{R}^d$ becomes isotropic in $\mathbb{R}^n$, where $n > d$ [1, 7, 3, 2]. Then, a uniform mesh in $\mathbb{R}^n$ will correspond to an anisotropic mesh in $\mathbb{R}^d$. The choice of the embedding map determines the resulting anisotropy of the mesh. For example, when the Gauss map of a surface is used, it will result a curvature adapted anisotropic surface mesh. On the other hand, we can choose the components of ∇f as the codimension of the embedding space, where $f : \Omega \subset \mathbb{R}^d \to \mathbb{R}$ is a function to be interpolated, the result will be an anisotropic mesh according to the gradient field of f.

We have implemented this approach using the standard mesh adaptation framework. Starting from an initial mesh of the mesh domain, we iteratively modify the mesh by the standard local mesh operations, flips, smoothing, vertex insertion and deletion. With the goal to make the mesh as uniform as possible in the embedding space. Results of adapted anisotropic surface and tetrahedral meshes as well as comparison with metric-based methods are presented.

References

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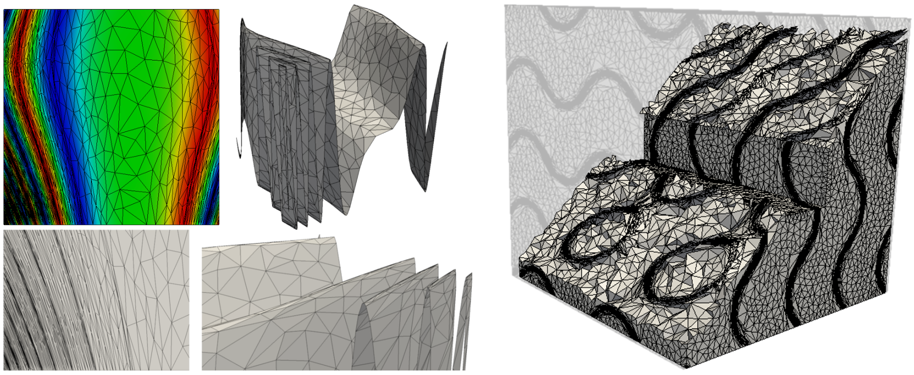

(a) (b)

Figure 1: Triangular anisotropic adapted mesh of a two dimensional function with internal layers (a). Cross section of a tetrahedral anisotropic adapted mesh of a three dimensional function with a complex internal layer, (b).

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