

Discrete Mathematics for Bioinformatics (P1)

WS 2009/10

Exercises 2

1. Affine Combinations

a) Given $x^0, x^1, \dots, x^k \in \mathbb{R}^n$ show that

$$\text{aff}(x^0, x^1, \dots, x^k) = \{x^0\} + \text{lin}(x^1 - x^0, \dots, x^k - x^0)$$

b) x^0, x^1, \dots, x^k are called *affinely independent* if the unique solution of

$$\sum_{i=0}^k \lambda_i x^i = 0, \quad \sum_{i=0}^k \lambda_i = 0$$

is $\lambda_i = 0$, for all $i = 0, \dots, k$.

How many affinely independent points exist in \mathbb{R}^n ?

2. Hulls

For the set of points

$$S = \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right\} \subset \mathbb{R}^3$$

give linear inequality descriptions of the polyhedra $\text{lin}(S)$, $\text{cone}(S)$, $\text{aff}(S)$, $\text{conv}(S)$.

3. Bases and Basic Solutions

Consider the polyhedron $P \subset \mathbb{R}^2$ defined by the system of linear inequalities

$$x_1 + x_2 \leq 2, \quad 0 \leq x_1 \leq 1, \quad 0 \leq x_2 \leq 2 \quad (*)$$

- Write (*) in the form $Ax \leq b$, for some $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$.
- Determine the bases of (*) and the corresponding basic solutions.
- Which of them are feasible?
- Give for each vertex of P the corresponding feasible bases.

4. Simplex Algorithm

A farmer owns 100 acres of land. He can cultivate potatoes or corn.

Given the data

	Potatoes	Corn	Available
Cultivation costs (in k€/a.)	1	2	110 k€
Working days per a.	1	4	160 working days
Profit (in k€/a.)	1	3	

what should he plant in order to maximize his income ?

- (a) Model the problem as a linear optimization problem.
- (b) Solve it by applying the Simplex method.