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Exercises 13

1. **Turing machine simulation**

   Given a Turing machine M accepting the language $L = \{0^n\,1^n \mid n \geq 1\}$ and the function δ:

δ	0	1	X	Y	#
q_0	(q_1, X, R)	–	–	(q_3, Y, R)	–
q_1	$(q_1, 0, R)$	(q_2, Y, L)	–	(q_1, Y, R)	–
q_2	$(q_2, 0, L)$	–	(q_0, X, R)	(q_2, Y, L)	–
q_3	–	–	–	(q_3, Y, R)	$(q_4, \#, R)$
q_4	–	–	–	–	–

   Simulate M on some accepted and some rejected inputs.

2. **Turing machine codes**

   Assume the Turing machine $M' = (\{q_1, q_2, q_3\}, \{0, 1\}, \{0, 1, \#\}, \delta, q_1, \#, \{q_2\})$ and the unary encoding:

   $0 \mapsto 0, 1 \mapsto 00, \# \mapsto 000, L \mapsto 0, R \mapsto 00.$

   Let M' have moves:

   - $\delta(q_1, 1) = (q_3, 0, R)$,
   - $\delta(q_3, 0) = (q_1, 1, R)$,
   - $\delta(q_3, 1) = (q_2, 0, R)$,
   - $\delta(q_3, \#) = (q_3, 1, L)$

   $\delta(q_i, X) = (q_j, Y, R)$ encoded by $\quad 0^i 1 \underbrace{0 \ldots 0}_{X} 10^j 1 \underbrace{0 \ldots 0}_{Y} 1 \underbrace{0 \ldots 0}_{R}$

   δ encoded by $\quad 111\,\mathtt{code}_1\,11\,\mathtt{code}_2\,11 \ldots 11\,\mathtt{code}_r\,111$

   Encoding of a Turing machine M denoted by $\langle M \rangle$.

   - **Generate** $\langle M' \rangle$

3. **Decision problems**

Consider:

- the general halting problem K: "Does Turing machine M_n halt for input i?" and
- the special halting problem K' "Does Turing machine M_n halt for input n?"

(a) Prove that K' is undecidable but semi-decidable.

(b) Use reduction to prove that K is undecidable.