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Discrete Mathematics for Bioinformatics (P1)

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Exercises 1

1. Sign up for the mailing list: *bioinf_P1_2009@lists.spline.inf.fu-berlin.de*

2. Transform the linear optimization problem

$$\begin{array}{rrcrcl} \min & 2x_1 & + & 3x_2 & & \\ \text{w.r.t.} & 3x_1 & + & 2x_2 & \leq & 7 \\ & 2x_1 & + & x_2 & = & 5 \\ & x_1 & & & \geq & 0 \end{array}$$

   to the standard form $\max\{c^T x \mid Ax = b, x \geq 0\}$.

3. Consider the linear optimization problem:

$$\begin{array}{rrcrcl} \max & 3x_1 & + & 4x_2 & & \\ \text{w.r.t.} & 3x_1 & + & 2x_2 & \leq & 12 \\ & 5x_1 & + & 10x_2 & \leq & 30 \\ & & & 2x_2 & \leq & 5 \\ & x_1, & & x_2 & \geq & 0 \end{array}$$

   (a) Determine the feasible region.
   (b) Solve the optimization problem graphically.
   (c) Solve the problem for the new objective function $6x_1 + 12x_2$.

4. Consider the linear optimization problem:

$$\begin{array}{rrcrcl} \max & c_1x_1 & + & c_2x_2 & & \\ \text{w.r.t.} & -x_1 & + & x_2 & \leq & 1 \\ & x_1, & & x_2 & \geq & 0 \end{array}$$

   Determine coefficients (c_1, c_2) of the objective function such that

   (a) the problem has a unique optimal solution.
   (b) the problem has multiple optimal solutions and the set of optimal solutions is bounded.
   (c) the problem has multiple optimal solutions and the set of optimal solutions is unbounded.

(d) the problem has feasible solutions, but no optimal solutions.

Finally, add one constraint so that the problem becomes infeasible.

5. **Diet Problem**
Suppose the only foods available in your local store are potatoes and steak. The decision about how much of each food to buy is to made entirely on dietary and economic considerations. We have the nutritional and cost information in the following table:

	Per unit of potatoes	Per unit of steak	Minimum requirements
Units of carbohydrates	3	1	8
Units of vitamins	4	3	19
Units of proteins	1	3	7
Units of fat	0	3	6
Costs per unit	25	50	

The problem is to find a diet (a choice of the numbers of units of the two foods) that meets all minimum nutritional requirements at minimal cost.

(a) Formulate the problem in terms of linear inequalities and an objective function.

(b) Solve the problem graphically.