# Errata

## to the third edition of

## A Concise Introduction to Mathematical Logic

**last modified December 1, 2010**

**Page 16**, line 2: $\neg\neg f$ ***instead of*** $\neg\neg f\vec{x}$

**Page 26**, line 17: (IS) ***instead of*** (MI)

**Page 34**, line 8: $a_\nu$ ***instead of*** $\alpha_n$

**Page 61**, line 10 from below: $x \notin \mathit{free}\,\varphi$ is indispensable for $\varphi\frac{t}{x} = \varphi$
***instead of*** these restrictions are indispensable

**Page 69**, line 12 from below: $\varphi$ ***instead of*** $\alpha$

**Page 77**, two corrections:
line 13: symbols $\neg$ or ( ***instead of*** symbols $\neg$ or $\wedge$ [1]
line 17: $\beta$ ***instead of*** $b$

**Page 78**, line 5: $\wedge$, $\vee$, $\forall$, and $\exists$ ***instead of*** $\wedge$, $\vee$, and $\exists$

**Page 90**, Exercise 6.1: $T$ should supposed to be consistent.

**Page 97**, line 16: to ***instead of*** too

**Page 103**, line 6: $\alpha$ ***instead of*** $\varphi$

**Page 110**, line 14:
$\exists x\varphi \wedge \exists y\forall x(\varphi \to x \leqslant y) \to \exists z\forall x[(\varphi \to x \leqslant z) \wedge \forall y(\forall x(\varphi \to x \leqslant y) \to z \leqslant y)]$
***instead of***
$\exists x\varphi \wedge \exists y\forall x(\varphi \to x \leqslant y) \to \exists z\forall x[(\varphi \to x \leqslant z) \wedge \forall y((\varphi \to x \leqslant y) \to z \leqslant y)]$

**Page 114**, line 4 from below: $b = d$ ***instead of*** $b = c$

[1] Replacing $\wedge$ by the left parenthesis makes the proof of Theorem **4.2** correct. The formula $\varphi := (\forall x x \equiv x \wedge y \equiv z)$ yields a counterexample to the original argument. Here is $s\varphi = 0$ according to the original text although $\varphi$ is not a PNF. The original definition of $s\varphi$ is appropriate only if $\varphi$ is written in Polish notation without parentheses, i.e., if $\wedge\forall x x = x\, y = z$ is written instead of $(\forall x x = x \wedge y = z)$. This remark concerns also former editions of the book.

**Page 120**, line 5: ***Insert*** and consistent ***before*** theory

**Page 122**, line 17: $\alpha \vDash \alpha\,\frac{y}{x}$ ***instead of*** $\alpha \vDash \frac{y}{x}$

**Page 134**, line 12: $y = 0 \wedge x = 0$ ***instead of*** $y = 0$

**Page 137**, line 17: *Delete the word* Choose

**Page 139**, line 4 from below: 2 ***instead of*** $n + 2$

**Page 141**, two corrections:
line 7: $\forall x \exists y (x \neq 0 \to x \cdot y = 1)$ ***instead of*** $\forall x \exists y (x \neq 0 \to x \cdot y = 0)$,
line 16 from below: ***Insert*** universal ***before*** Horn formula.

**Page 148**, two corrections:
line 2 from below: CNF ***instead of*** KNF, last line footnote: $\Lambda^1$ ***instead of*** $\Lambda^n$.

**Page 157**, line 2 from below: $\{\neg\beta_0, \ldots, \neg\beta_m\}$ ***instead of*** $\{\neg\beta_1, \ldots, \neg\beta_n\}$

**Page 159**, two corrections: line 14: **4.4** ***instead of*** **4.3**,
line 14 from below: $\mathcal{P} \vdash \mathsf{sum}\,\underline{n}\,\underline{1}\mathsf{S}\underline{n}$ ***instead of*** $\mathcal{P} \vdash \mathsf{sum}\,\underline{n}, \mathsf{S}\underline{n}$

**Page 162** In the proof of Lemma **6.1** three times: $H_0^\omega$ ***instead of*** $H_0^{\rho\omega}$.

**Page 165**, line 6 from below: $\mathcal{P}_g$ ***instead of*** $\mathcal{P}_{g_i}$

**Page 168**, three corrections:
line 8: $\mathcal{P} \vdash \gamma^\omega$ ***instead of*** $\beta^{\rho_0\omega} = \gamma_i^\omega$
lines 10+11 in all six occurrences: $\sigma$ ***instead of*** $\tau$
line 13 from below: $\sigma_1 := \sigma$ ***instead of*** $\sigma_1 := \tau$

**Page 171**, two corrections:
line 16: only if ***instead of*** iff
line 19: By definition, ***instead of*** and also the converse of the 1st claim is obvious.

**Page 172**, line 16: ***insert*** sentence ***before*** $\alpha \in \mathcal{L}A$

**Page 176**, two corrections: line 8: only if ***instead of*** iff
line 16: if $T$ is any theory in $\mathcal{L}$ ***instead of*** if $\mathcal{A} \vDash T$

**Page 184**, line 13: $\geqslant 2^{k-2}$ ***instead of*** $\geqslant 2^{k-2}-1$

**Page 186**, line 8: $\alpha$ ***instead of*** $\beta$

**Page 188**, four corrections in the proof of Theorem **4.4** as follows:
First line: $T$ is inconsistent, for then $T$ is axiomatized by $\forall x\, x \neq x$.
***instead of*** $\vdash_T \forall xy\, x = y$, for then $T$ is axiomatized by $\forall xy\, x = y$.
line 6: set $I$ ***instead of*** $I \neq \emptyset$
line 8: Let first $I \neq \emptyset$ and choose ***instead of*** Choose, ***Add to last line***
The case $I = \emptyset$ is treated more easily. Instead of $\binom{*}{*}$ prove $P \nvdash_T \bot$ ($\equiv \boldsymbol{v}_0 \neq \boldsymbol{v}_0$) similarly to $P \nvdash_T \pi_i$ above. This clearly confirms $(*)$.

**Page 203**, line 3:
then meets some specific problems ***instead of*** is not obliged to do so
(see e.g. Poizat, A course in model theory, pp 60-61)

**Page 219**, two corrections:
line 8: $\mathrm{I}_2^2$ ***instead of*** $\mathrm{I}_1^2$, line 6 from below: $\sum$ ***instead of*** $\prod$

**Page 220** in the first paragraph: sg ***instead of*** $\sigma$

**Page 224**, line 3: ***remove*** *pd*

**Page 226**, line 8 from below: $f : \mathbb{N} \to \mathbb{N}_+$ ***instead of*** $f \in \mathbf{F}_1$

**Page 230**, two corrections:
line 15: $k{<}\ell b$ ***instead of*** $k{<}b$, line 18: **2.2** ***instead of*** **2.3**

**Page 234**, line 13: $\tilde{\forall}(b, c)$ ***instead of*** $\tilde{\forall}(b, c, d)$

**Page 245**, line 1: $\mathrm{lcm}\{d_\nu \mid \nu{<}k\}$ ***instead of*** $\mathrm{lcm}\{d_\kappa \mid \nu{<}k\}$

**Page 249**, two corrections: line 10: $\underline{b}$ ***instead of*** $\mathtt{S}\underline{b}$
line 7 from below: $(\alpha \wedge \beta)$ ***instead of*** $(\alpha \tilde{\wedge} \beta)$

**Page 252**, last line: $\mathtt{bwb}(x) \equiv_{\mathcal{N}} \mathtt{prov}(x)$ ***instead of*** $\mathtt{bew}(y, x) \equiv_{\mathcal{N}} \mathtt{prov}(y, x)$

**Page 253**, line 15: $\forall x \varphi$ ***instead of*** $\varphi(x)$

**Page 256**, line 12: unprovable ***instead of*** provable

**Page 257**, line 7 from below: undecidable ***instead of*** decidable

**Page 258**, third line from below: $\alpha^{\mathsf{P}} \equiv_\Delta \alpha$ ***instead of*** $\alpha^{\mathsf{P}} \equiv \alpha$

**Page 259**, two corrections:
line 4: *CA* ***instead of*** $\Delta$, line 2 from below: Exercise 4 ***instead of*** Exercise 3

**Page 261**, two corrections:
line 3: $(\varphi(x) \to \varphi(x \cup \{y\})) \to \forall x \varphi(x)$ ***instead of*** $(\varphi(x) \to \varphi(x \cup \{y\}) \to \forall x \varphi(x))$
line 4: $\exists x($ ***instead of*** $\exists($

**Page 262**, line 13 from below: Lemma **5.3** ***instead of*** Theorem **5.4**

**Page 265**, line 9: $\neg\, bwb((0 \neq 0)^{\cdot})$ ***instead of*** $\neg\, bwb(\ulcorner \emptyset \neq \emptyset \urcorner)$

**Page 267**, line 17: $\mathrm{sb}(x, y)$ ***instead of*** $\mathrm{sb}(u, y)$

**Page 274**, five corrections:
line 10: $\mathsf{c} : \mathbb{N} \to \mathbb{N}_+$ ***instead of*** $\mathsf{c}$
line 11: $\mathsf{d}_0, \ldots, \mathsf{d}_n$ in $\mathbb{N}_+$. ***instead of*** $\mathsf{d}_0, \ldots, \mathsf{d}_n$.
line 14: $(\forall \nu \leqslant x) \mathsf{d}_\nu \,|\, y \wedge y \neq 0 \wedge (\forall z < y)(z \neq 0 \to (\exists \nu \leqslant x)\, \mathsf{d}_\nu \not| \; z)$
***instead of*** $(\forall \nu \leqslant x) \mathsf{d}_\nu \,|\, y \wedge (\forall z < y)(\exists \nu \leqslant x)\, \mathsf{d}_\nu \not| \; z$
line 16 from below: $\mathsf{d}_0, \ldots, \mathsf{d}_x$ ***instead of*** $\mathsf{c}_0, \ldots, \mathsf{c}_x$
line 13 from below: **6.4** ***instead of*** **6.2**

**Page 275**, two corrections:
line 4: $\underline{f(\vec{a}, b)}$ ***instead of*** $\underline{f\vec{a}}$, line 15: $\nu \leqslant y$ ***instead of*** $\nu \leqslant \mathsf{S}y$

**Page 276**, line 4: (6) ***instead of*** (3)

**Page 277**, two corrections:
line 12 from below: $\mathsf{S}x$ ***instead of*** $\mathsf{S}y$, last line: $\forall \vec{x} \alpha$ ***instead of*** $\forall \vec{x}$

**Page 292**, three corrections:
line 3: $\Pi_2$ ***instead of*** $\Pi_1$, line 14: $\forall x \varphi(x) \in \mathcal{L}^0_{ar}$ ***instead of*** $\varphi \in \mathcal{L}^1_{ar}$,
line 4 from below: $\exists y \forall x \gamma$ ***instead of*** $\exists z \forall x \gamma$

**Page 296**, line 3 from below: $P \Vdash \Box p \wedge \neg \Box q$ ***instead of*** $P \Vdash \Box p \wedge \neg \Diamond q$

**Page 298**, line 2 from below: $\vdash_{\mathsf{Gj}} H$ ***instead of*** $\vdash_{\mathsf{Gj}}$