Exercise 1:
Prove the following lemmas from the lecture.

Lemma.
Let $G = (V, E, H, I)$ be a SEAG with $n$ alignment edges and $m$ interaction matches. Then

1. $P_R(G)$ is full-dimensional and
2. the inequality $x_i \leq 1$ is facet-defining iff there is no $e_j \in E$ in conflict with $e_i$.

Lemma.
Let $G = (V, E, H, I)$ be a SEAG with $n$ alignment edges and $m$ interaction matches.

1. The inequality $x_i \geq 0$ is facet-defining iff $e_i$ is not contained in an interaction match.
2. For each interaction match $m_{i,j}$ the inequality $y_{ij} \geq 0$ is facet-defining.

Exercise 2:
Prove that the number of clique inequalities arising from a complete bipartite graph $K_{p,q}$ is $(\binom{p+q-2}{p-1})$.

Hint: Develop a recurrence relation for the number of source-sink paths in $PG(K_{p,q})$ and use complete induction to prove the statement.

Exercise 3:
Consider the problem of tiling a planar region $R$ with $n$ dominoes. Each domino is a $2 \times 1$ rectangle. $R$ is an arbitrary collection of $2n$ $1 \times 1$ squares. Figure 1 shows one example of such a region. The squares are numbered 1 through $2n$. $R$ is described by the set of all the pairs $(a, b)$, $a, b \in \{1, 2, \ldots, 2n\}$, $a < b$, such that square $a$ and square $b$ are edge-connected (i.e., have an edge in common). In this representation, let $R = \{p_1, p_2, \ldots, p_r\}$, where each $p_k$, for $k = 1$ to $r$, is a pair of edge-connected squares.
Abbildung 1: Example of a region R where \( 2n = 16 \). A description of \( R \) is the set 
\[ \{(1,2), (2,3), (3,4), (2,5), \ldots, (14,16), (15,16)\} \]

\[ a) \] Model the problem as a constraint satisfaction problem where the dominoes are the variables, that is, define the variable domains and the constraints.

\[ b) \] Model the problem as a constraint satisfaction problem where the squares are the variables.

Exercise 4:
Consider the following network

Assume that each variable (node) has a domain of \( \{1,2,3,4\} \).

\[ a) \] Model the problem as a constraint satisfaction problem

\[ b) \] Apply arc consistency to reduce the domains of the variables.

\[ c) \] What further reduction can be obtained by fixing the value of the node 5 to the minimum possible value?