1 Forward algorithm

Compute the probability $Pr(X)$ to observe the sequence $X = xxy$ using the Forward algorithm.

Forward algorithm:

Input:
HMM $M = (\sum, Q, A, e)$ and symbol sequence $X$

Output:
Probability $Pr(X)$

Initialization ($i = 0$):
$F(0, 0) = 1$, $F(s, 0) = 0$ for $s \in Q \setminus \{0\}$

Recurrence:
for all $i = 1 \ldots |X|$, $t \in Q$:
$F(t, i) = e_t(x_i) \sum_{s \in Q} F(s, i-1) a_{st}$

Termination ($i = |X| + 1$):
$Pr(x) = \sum_{s \in Q} F(s, |X|) a_{s,0}$

2 Backward algorithm

Compute the probability $Pr(X)$ to observe the sequence $X = xxy$ using the Backward algorithm.

Backward algorithm:
Input: HMM $M = (\Sigma, Q, A, e)$ and symbol sequence $X$

Output: Probability $\Pr(X)$

Initialization ($i = |X|$):
$B(s, |X|) = a_{s,0}$ for all $s \in Q$

Recurrence:
for all $i = |X| - 1 \ldots 1$, $s \in Q$:
$B(s, i) = \sum_{t \in Q} a_{st} e_t(x_{i+1})B(t, i + 1)$

Termination ($i = 0$):
$\Pr(x) = \sum_{t \in Q} a_{0,t} e_t(x_1)B(t, 1)$

3 Posterior Decoding

Compare your results from exercise 1 and 2.

Determine the path of states $\hat{\pi}$ for the sequence $X = xxy$ by posterior decoding.

$\hat{\pi}_i = \arg \max_{s \in Q} \frac{\Pr(\pi_i = s, x)}{\Pr(X)}$

with $\Pr(\pi_i = s, x) = F(s, i)B(s, i)$