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# Optimization

WS 2015/16

## Exercises 5

### 1. Lagrangean Relaxation I

Consider the following problem

$$\begin{array}{llll} \min & 2x_1 & - & 3x_2 \\ \text{w.r.t.} & 3x_1 & - & 4x_2 \leq -6 \\ & -x_1 & + & x_2 \leq 2 \\ & 6x_1 & + & 2x_2 \geq 3 \\ & 6x_1 & + & x_2 \leq 15 \\ & & & x_1, x_2 \geq 0 \\ & & & x_1, x_2 \in \mathbb{Z} \end{array}$$

- Draw the corresponding polytope and determine graphically the optimal solution  $Z_{IP}$  of the original problem and  $Z_{LP}$ , the solution of the LP-relaxation.
- Now apply lagrangean relaxation by relaxing the first inequality. Draw the polytope of the relaxed ILP. Determine the set  $X$  of feasible solutions for the relaxed problem.
- The new objective function is then:

$$Z(P) = \min_{(x_1, x_2) \in X} 2x_1 - 3x_2 + p(6 + 3x_1 - 4x_2)$$

Calculate  $Z_D = \max_{p \geq 0} Z(p)$  and compare this value to  $Z_{IP}$  and  $Z_{LP}$ . (To obtain  $Z_D$ , draw the graphs of the function  $f(p) = 2x_1 - 3x_2 + p(-6 - 3x_1 + 4x_2)$  for all  $(x_1, x_2) \in X$ . Do this by hand.)

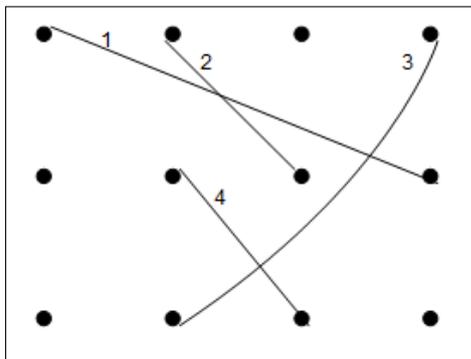
- repeat a-c for the objective functions  $-x_1 + x_2$  and  $-x_1 - x_2$  and compare  $Z_{LP}$ ,  $Z_D$ , and  $Z_{IP}$ . To draw the graphs of the functions  $f(p)$  use MATLAB.

## 2. Lagrangean Relaxation II

Prove Lemma 1 (see script page 4001) stating that (in case of a minimization problem) if  $\lambda \geq 0$ , then  $Z(\lambda) \leq Z_{IP}$ , where  $Z_{IP}$  is the optimal value of an original ILP and  $Z(\lambda)$  is the optimal value of the relaxed problem for a given value of the Lagrangean multiplier  $\lambda$ .

### 3. Branch and Cut

Given the following alignment graph:



All edges have weight 1.

- Try to solve the alignment problem by using branch-and-cut: Add mixed cycle inequalities (see the ‘shortest path’ method in the script, page 18) to the corresponding (relaxed) LP. Can you reach an optimal solution for the ILP without branching?
- Now use branching to solve the problem.
- Instead of branching, just add the inequality

$$x_1 + x_2 + x_3 + x_4 \leq 2$$

Can you solve the ILP now?

- Prove that the inequality in (c) is facet-defining.