A sharp interface finite volume method for for variable density zero Mach number two–phase flow with surfactant-dependent surface tension

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I. Flow Solver based on SAMRAI
(see https://computation.llnl.gov/casc/SAMRAI/)

II. Sharp interface discretizations
II.a) Pressure field and pressure jump force

II.b) Surface tension – well-balanced
An established balancing strategy for stiff forces, [4, 5], repre-
sents the force as the discretized gradient of an auxiliary hydro-
static pressure. This allows balancing up to machine accuracy,
but is non-conservative in general, see proposal.

II.c) Surface tension – conservative and well-balanced
Piecewise circular reconstruction yields unique tanguents, 0, and
curvatures, i.e., at interface-cell intersections. Net sur-
factance force on C1 can be expressed identically using the
vector of its normal:

\[ P(\theta) = \alpha(x) + \beta(x) \]

Flow field after one time step for an under-resolved unit-circle un-
der surface tension \( \sigma \) with fluid density \( \rho = 1 \). Levelset data correspond to exact dis-
tance function on cell centers. This allows us to use the con-
formal form of all branches. For details, see [6].

III. Projection step for density ratios \( \frac{\rho_2}{\rho_1} \leq \epsilon < 1 \)
In a liquid gas flow, gas phase pressure perturbations are \( \Delta \rho / \rho \) relative to those in the liquid, while both phases attain \( \Delta \rho \) velocity magnitude. Therefore,

\[ \Delta \rho = 0 \quad \Rightarrow \quad \Delta \rho / \rho = 0 \]

Let

\[ \rho_2(x) = \rho_2^n(x) + \Delta \rho_n(x) \]

Leading-order Problem, liquid phase

\[ \rho_2^n \frac{\partial P}{\partial n} = \frac{\partial}{\partial n} \left( \frac{\rho_2^n}{\rho_2^n + \rho_1} \right) = 0 \]

Leading-order result, gas phase

\[ \rho_2^n = \rho_2^n - \rho_1 = \rho_1 \rho_2^n \]

Coupling conditions

\[ \rho_2^n = \rho_2^n = \rho_1 \quad \Rightarrow \quad \rho_2^n = \rho_1 \]

1D test of the “asymptotic preconditioner” left: perturbation and exact solutions (virtually identical for all values of \( \epsilon \)); right: cond-
tion numbers.

IV. Levelset-VOF-hybrid advection scheme
Coupled Levelset-VOF scheme derived from [6] –

References
[6] Schneider T., Klein R., Overcoming Mass Losses in Level Set-Based Arti-
[7] S. Rusth, B. Merwe, A single embedding method for solving partial differ-
[8] R. Ihlenfeld, A finite volume implementation of an embedding method for solv-