1. **Bin Packing**

Consider the following variant of the *bin packing* problem:

- Pack \( n \) items of size \( g_i, i = 1, \ldots, n \), into (at most) \( n \) bins, each of capacity \( c \).
- Put the first \( m \) items into different bins.
- Find the minimal number of bins necessary.

Model the problem in integer linear programming.

2. **IP Formulations**

Suppose that you are interested in choosing a set of investments \( \{1, \ldots, 7\} \) using \( 0 - 1 \) variables. Model the following constraints:

(a) You cannot invest in all of them.

(b) You must choose at least one of them.

(c) Investment 1 cannot be chosen if investment 3 is chosen.

(d) Investment 4 can be chosen only if investment 2 is also chosen.

(e) You must choose either both investments 1 and 5 or neither.

(f) You must choose either at least one of the investments 1, 2, 3 or at least two investments from 2, 4, 5, 6.

3. **n-Queens Problem**

Model the \( n \)-queens problem (as an integer linear program):

Place \( n \) queens on an \( n \times n \) chess board such that in each line (horizontal, vertical and diagonal) only one queen is allowed.
4. **SCIP**

Use SCIP to solve the following exercise:

There are 3 depots and 4 customers and each customer ordered 1 package.

$f_i$ denotes the costs to open the depot $i$, $c_{ij}$ are the costs for delivering the package from depot $i$ to the customer $j$.

Each customer has to get his package and the aim is to minimize the costs. The given values are: $f_1 = 3$, $f_2 = 2$, $f_3 = 4$ and

| $c_{11}$ = 2 | $c_{21}$ = 3 | $c_{31}$ = 1.5 |
| $c_{12}$ = 2.5 | $c_{22}$ = 4.5 | $c_{32}$ = 1 |
| $c_{13}$ = 2 | $c_{23}$ = 4.5 | $c_{33}$ = 1.5 |
| $c_{14}$ = 3 | $c_{24}$ = 5 | $c_{34}$ = 2 |

Of course customer $j$ can only get his package from depot $i$ if this depot is open. Thus:

$x_{ij} \leq y_i$, where

$y_i = \begin{cases} 
1, & \text{depot } i \text{ is open} \\
0, & \text{else}
\end{cases}$

Don’t use the command “Binary” but Bounds (≥ 0 and ≤ 1) for the variables $x_{ij}$ and $y_i$. Formulate the problem in two different ways and compare the results:

(a) $x_{ij} \leq y_i$ for every $j$ and $i$.

(b) rewrite the above formulation such that $\sum_{j=1}^{4} x_{ij} \leq 4y_i$ for all $i = \{1, 2, 3\}$.