1. Bellman-Ford (Niveau I)

(a) Use the Bellman-Ford algorithm to determine the shortest path from source $A$ to any other node in the graph.

(b) Let $D = (V, A), n = |V|$ be a directed graph. Prove that $D$ contains a circuit of negative length reachable from $s$ if and only if $f_n(v) \neq f_{n-1}(v)$, for some $v \in V$, where $f_k(v) = \min\{l(P) | P$ is an $s - v$ walk traversing at most $k$ arcs$\}$

![Graph with weights](image)

2. Network Flow (Niveau II) Prove the Theorem:

For a network $(V, E, s, t)$ with capacities $\text{cap} : E \rightarrow \mathbb{R}^+$ the maximum value of a flow is equal to the minimum capacity of an $(s, t)$-cut:

$$\max\{\text{val}(f) \mid f \text{ is a flow}\} = \min\{\text{cap}(S, T) \mid (S, T) \text{ is an } (s, t)\text{-cut}\}$$

Hint: Show that the following conditions are equivalent:

(a) $f$ is a maximum flow.

(b) The residual network $G_f$ contains no augmenting path.

(c) $\text{val}(f) = \text{cap}(S, T)$ for some cut $(S, T)$ of $G$
3. **Ford-Fulkerson (Niveau I)**

(a) Use the Ford-Fulkerson algorithm to find a maximum flow in the network.

Start with the initial flow $f$. An edge label $f/c$ means initial flow $f$ and capacity $c$.

(b) Find a minimum cut proving the maximality of the flow.

4. **Matching and Bipartite Graphs (Niveau I)**

(a) Apply the matching augmenting algorithm for bipartite graphs to the graph below and compute a maximum cardinality matching from the initial matching.