Reducing 3SAT to INDEPENDENT SET

- Let $F$ be a conjunction of $n$ clauses of length 3, i.e., a disjunction of 3 propositional variables or their negation.
- Construct a graph $G$ with $3n$ vertices that correspond to the variables in $F$.
- For any clause in $F$, connect by three edges the corresponding vertices in $G$.
- Connect all pairs of vertices corresponding to a variable $x$ and its negation $\neg x$.
- $F$ is satisfiable if and only if $G$ contains an independent set of size $n$.

Solving numerical constraints

<table>
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<tr>
<th>Satisfiability</th>
<th>over $\mathbb{Q}$</th>
<th>over $\mathbb{Z}$</th>
<th>over $\mathbb{N}$</th>
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<tbody>
<tr>
<td>Linear equations</td>
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<td>decidable</td>
<td>undecidable</td>
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NP-hard problems

- Decision problem: solution is either yes or no
- Example: Traveling salesman decision problem:
  Given a network of cities, distances, and a number $B$, does there exist a tour with length $\leq B$?
- Search problem: find an object with required properties
- Example: Traveling salesman optimization problem:
  Given a network of cities and distances, find a shortest tour.
- Decision problem $NP$-complete $\Rightarrow$ search problem $NP$-hard
- $NP$-hard problems: at least as hard as $NP$-complete problems

Graph theoretical problems

- Shortest path $\text{polynomial}$
- Traveling salesman $NP$-hard
- Minimum spanning tree $\text{polynomial}$
- Steiner tree $NP$-hard
NP-hard problems in bioinformatics

- Multiple sequence alignment
  Wang/Jiang 94
- Protein folding
  Fraenkel 93
- Protein threading
  Lathrop 94
- Protein design
  Pierce/Winterf 02
- ...

Further complexity classes

coNP: Problems whose complement is in NP
PSPACE: Problems solvable in polynomial space
EXPTIME: Problems solvable in exponential time

Literature

- J. E. Hopcroft and J. D. Ullman: Introduction to automata theory, languages and computation. Addison-Wesley, 1979
- C. H. Papadimitriou: Computational complexity. Addison-Wesley, 1994