Discrete Mathematics for Bioinformatics (P1)  
WS 2010/11  
Exercises 3

1. **Skip lists (Niveau I)**  
   Compute the expected value for the height \( h \), search time and space consumption if the probability \( p \) for each coin flip to produce a 1 is \( \frac{1}{3} \).

2. **”sparse” skip list (Niveau I)**  
   Each node in the skip list has up to two incoming directed edges pointing to other nodes in the skip list.
   
   (a) Which edges are really necessary for a search and which can be removed?  
   (b) Can you give a rough estimate for the expected number of edges that can be removed?

3. **Skip lists (Niveau II)** Proof that the height of a skip list has expected value \( O(\log n) \) with high probability.  
   Hint: You do not need Chernoff bounds or Markov’s inequality to show this.

4. **Tail estimates (Niveau II)**  
   Show that randomized Quicksort has expected run time \( O(n \log n) \) with high probability.  
   Hint: Use the idea of successful/ unsuccessful splits. Let a split be successful if it splits the total set into two fractions such that both contain less than 75% of the complete set. What are the probabilities of these splits and how many are required to sort?