1. SAT Problem (NIVEAU I)
   The pigeon-hole SAT problem expresses the problem of finding a way to place \( n \) pigeons
   in \( n - 1 \) pigeon-holes such that no hole contains more than one pigeon. Obviously, this
   problem is unsatisfiable.
   
   • Model the Pigeon-hole SAT problem. (See script: Literals, clauses, clause-sets)

2. Turing machine simulation (NIVEAU I)
   Given a Turing machine \( M \) accepting the language \( L = \{0^n 1^n \mid n \geq 1\} \) with accepting
   state \( q_4 \) and the next move function \( \delta \):

   \[
   \begin{array}{c|cccc}
   \delta & 0 & 1 & X & Y & \# \\
   \hline
   q_0 & (q_1, X, R) & - & - & (q_3, Y, R) & - \\
   q_1 & (q_1, 0, R) & (q_2, Y, L) & - & (q_1, Y, R) & - \\
   q_2 & (q_2, 0, L) & - & (q_0, X, R) & (q_2, Y, L) & - \\
   q_3 & - & - & - & (q_3, Y, R) & (q_4, \#, R) \\
   q_4 & - & - & - & - & - \\
   \end{array}
   \]

   Simulate \( M \) on input 0011 and 001101.

3. Decision problems (NIVEAU II)
   Let \( w_i \) be the \( i \)-th word in \( \{0, 1\}^* \) and \( M_n \) the \( n \)-th turing machine. Consider:
   
   • the general halting problem \( K \): “Does Turing machine \( M_n \) halt for input \( w_i \)”
     
     and
   
   • the special halting problem \( K' \) “Does Turing machine \( M_n \) halt for input \( w_n \)”

   (a) Prove that \( K' \) is undecidable but semi-decidable.
   (b) Use reduction to prove that \( K \) is undecidable.