Prof. Dr. Knut Reinert,
Prof. Dr. Alexander Bockmayr,
René Rahn,
Annika Röhl

November 6, 2014

Algorithms

WS 2014/15

Exercises 4

1. **Modulo Arithmetic (Niveau II)**

   Prove the following theorem:
   For any positive intergers a and n, if $d = gcd(a, n)$ (the greatest common divisor of a and n), then
   $$\langle a \rangle = \langle d \rangle = \{0, d, 2d, \dots, n - d\}$$
   and thus
   $$|\langle a \rangle| = n/d$$
   ($\langle a \rangle := \{a \cdot i \mod n \mid i \in \mathbb{N}\}$).
   Hint: Use Bezout's lemma. It states that if a and b are nonzero integers with greatest common divisor d, then there exist integers x and y such that $ax + by = d$

2. **Hashing (Niveau I)**

   Consider a version of the division method in which $h(k) = k \mod m$, where m=$2^p - 1$ and k is a character string interpreted in radix 2^p. Show that if string x can be derived from string y by permuting its characters, then x and y hash to the same value.

3. **Hashing (Niveau I)**

   Consider the two situations in a hash table of size m using open addressing with linear probing:

   - You have $n = m/2$ keys in the table, where every even-indexed slot is occupied and every odd- indexed slot is free.
   - You have $n = m/2$ keys in the table and the first $n = m/2$ locations are the ones occupied.

   (a) Compute the average search cost for an unsuccessful search for both situations under the hypothesis of simple uniform hashing.

4. **Skip lists (Niveau I)**

   Compute the expected value for the height (h), search time and space consumption if the probability p for each coin flip to produce a 1 is 1/3.