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# Algorithms

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### Exercises 5

1. **Tree decomposition (Niveau I)**

   How large is the largest piece of a any tree decomposition for a graph $G$ of $n$ nodes if $G$ is a clique? Prove your answer.

2. **Tree decomposition (Niveau I)**

   Prove the following theorem:
   Let $G = (V, E)$ be a graph, $T$ be a tree decomposition of $G$, and $(x, y)$ an edge in $T$. The deletion of $(x, y)$ divides $T$ into two components $X$ and $Y$. Let $V_x$ and $V_y$ be the 'pieces' of $x$ and $y$, respectively. Then deleting the set $V_x \cap V_y$ from $V$ disconnects $G$ into the two subgraphs $G_X - (V_x \cap V_y)$ and $G_Y - (V_x \cap V_y)$.
   ($G_M$ for $M = X, Y$ is the subgraph of $G$ that consists of all nodes in the 'pieces' of $M$.)

3. **Tree decomposition (Niveau II)** Prove the following theorem:
   If graph G contains a $(w + 1)$-linked set of size at least $3w$, then G has tree-width at least $w$.

   Suppose, by way of contradiction, that $G$ has a $(w + 1)$-linked set $X$ of size at least $3w$, and it also has a nonredundant TD $(T; \{V_t\})$ of width less than $w$. The idea of the proof is to find a piece $V_t$ that is "centered" with respect to $X$, so that when some part of $V_t$ is deleted from $G$, one small subset of $X$ is separated from another.

4. **Tree decomposition (Niveau I)** Use the algorithm presented in the lecture to compute a tree decomposition of the graph below: