1. **Skip lists (Niveau I)**
   Compute the expected value for the height \( h \), search time and space consumption if the probability \( p \) for each coin flip to produce a 1 is 1/3.

2. **"sparse" skip list (Niveau I)**
   Each node in the skip list has up to two incoming directed edges pointing to other nodes in the skip list.
   
   (a) Which edges are really necessary for a search and which can be removed?
   (b) Can you give a rough estimate for the expected number of edges that can be removed?

3. **Skip lists (Niveau II)** Proof that the height of a skip list has expected value \( O(\log n) \) with high probability.
   
   Hint: You do not need Chernoff bounds or Markov’s inequality to show this.
4. Independencies

Random variables \((X_i)_{i \geq 1}\) are called *pairwise independent* if for all \(1 \leq i < j\) and all \(r_i\) and \(r_j\) holds:

\[
Pr(X_i = r_i \land X_j = r_j) = Pr(X_i = r_i) \cdot Pr(X_j = s_j)
\]

Random variables are called *mutual independent* if for all \(n \geq 2\) and all \(1 \leq i_1 < i_2 < \ldots < i_n\) and \(r_1, r_2, \ldots, r_n\) holds:

\[
Pr(\bigwedge_{k=1}^{n} (X_{i_k} = r_k)) = \prod_{k=1}^{n} Pr(X_{i_k} = r_k)
\]

(a) Let \(X\) and \(Y\) be random variables:

i. Prove that \(E(X + Y) = E(X) + E(Y)\).

ii. Assume that \(X\) and \(Y\) are independent. Prove that \(E(XY) = E(X)E(Y)\)

(b) Given the sample space:

\[
U = \{(123), (132), (213), (231), (312), (321), (111), (222), (333)\}
\]

We choose a random element \(u\) in \(U\). Let \(X_i\) the digit in \(u\) at position \(i\) (for \(i = 1, 2, 3\)), e.g. \(X_3 = 2\) for \(u = (312)\). Let \(N\) the random variable that equals \(X_2\). Prove:

i. \(\forall i, r: 1 \leq i \leq 3, 1 \leq r \leq 3\) gilt: \(Pr(X_i = r) = \frac{1}{3}\).

ii. \(X_1, X_2,\) and \(X_3\) pairwise independent.

iii. \(X_1, X_2,\) and \(X_3\) are not mutual independent.

iv. \(E(N) = 2\).

v. \(\sum_{i=1}^{E(N)} E(X_i) = 4\).

vi. \(E(\sum_{i=1}^{N} X_i) \neq \sum_{i=1}^{E(N)} E(X_i)\).