Algorithms

WS 2012/13

Exercises 2

1. **Network Flow (Niveau II)** Prove the Theorem:
   For a network \((V, E, s, t)\) with capacities \(\text{cap} : E \to \mathbb{R}_+\) the maximum value of a flow is equal to the minimum capacity of an \((s, t)\)-cut:
   \[
   \max\{\text{val}(f) \mid f \text{ is a flow}\} = \min\{\text{cap}(S, T) \mid (S, T) \text{ is a } (s, t)\text{-cut}\}
   \]

   Hint: Show that the following conditions are equivalent:
   
   (a) \(f\) is a maximum flow.
   (b) The residual network \(G_f\) contains no augmenting path.
   (c) \(\text{val}(f) = \text{cap}(S, T)\) for some cut \((S, T)\) of \(G\)

2. **Network Flow (Niveau I)** Assume a flow network with edge and additional vertex capacities. Each vertex \(v\) has a limit on the flow that can pass through it. Explain how to transform this flow network into an equivalent flow network without vertex capacities.

3. **Ford-Fulkerson (Niveau I)**
   
   (a) Use the Ford-Fulkerson algorithm to find a maximum flow in the network

   ![Network Diagram]

   Start with the initial flow \(f\). An edge label \(f/c\) means initial flow \(f\) and capacity \(c\).

   (b) Find a minimum cut proving the maximality of the flow.
4. Matching and Bipartite Graphs (Niveau I)

(a) Apply the matching augmenting algorithm for bipartite graphs to the graph below and compute a maximum cardinality matching from the initial matching.