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# Algorithms

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### Exercises 2

1. **Network Flow (Niveau II)** Prove the Theorem:
   For a network $(V, E, s, t)$ with capacities $\text{cap} : E \to \mathbb{R}_+$ the maximum value of a flow is equal to the minimum capacity of an $(s,t)$-cut:

   $$\max\{\text{val}(f) \mid f \text{ is a flow}\} = \min\{\text{cap}(S,T) \mid (S,T) \text{ is an } (s,t)\text{-cut}\}$$

   Hint: Show that the following conditions are equivalent:

   (a) $f$ is a maximum flow.

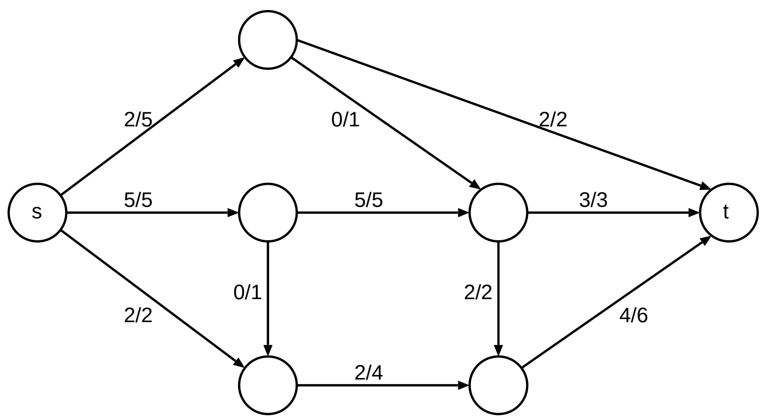
   (b) The residual network $G_f$ contains no augmenting path.

   (c) $\text{val}(f) = \text{cap}(S,T)$ for some cut $(S,T)$ of $G$

2. **Network Flow (Niveau I)** Assume a flow network with edge and additional vertex capacities. Each vertex $v$ has a limit on the flow that can pass through it. Explain how to transform this flow network into an equivalent flow network without vertex capacities.

3. **Ford-Fulkerson (Niveau I)**

   (a) Use the Ford-Fulkerson algorithm to find a maximum flow in the network

   Start with the initial flow $f$. An edge label $f/c$ means initial flow $f$ and capacity $c$.

   (b) Find a minimum cut proving the maximality of the flow.

4. **Matching and Bipartite Graphs (Niveau I)**

(a) Apply the matching augmenting algorithm for bipartite graphs to the graph below and compute a maximum cardinality matching from the initial matching.