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Algorithms

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Exercises 1

1. **MST - Approximation (Niveau I)**

   (a) Construct a complete graph with at least 6 nodes that satisfies the triangle inequality and apply the MST- approximation algorithm to approximate the optimal solution of the TSP.

   (b) Prove that the MST-approximation is a 2-approximation for the TSP.

2. **Amortized Analysis (Niveau I)**
   Assume an array of a certain initial size n. After n insertions the array if full and one approach is to allocate a bigger array and copy all previously inserted elements into the new array. The cost for insertion and copy of an element is $O(1)$ each.

   (a) How would you choose the size of the new array if you have to allocate additional space to achieve amortized linear runtime?

   (b) Use the potential method and accounting method to show that the amortized cost is indeed linear.

3. **Analysis of SELECTION algorithm (Niveau II)**
   Read the additional PDF document from Cormen and solve Exercise 9.3-1

4. **Bellman-Ford (Niveau I)**

   (a) Use the Bellman-Ford algorithm to determine the shortest path from source z to any other node in the graph.

   (b) Let $D = (V, A), n = |V|$ be a directed graph. Prove that D contains a circuit of negative length reachable from s if and only if $f_n(v) \neq f_{n-1}(v)$, for some $v \in V$, where $f_k(v) = \min\{l(P) | P \text{ is an } s - v \text{ walk traversing at most } k \text{ arcs}\}$