## Reducing 3SAT to INDEPENDENT SET

- Let $F$ be a conjunction of $n$ *clauses* of length 3, i.e., a disjunction of 3 propositional variables or their negation.
- Construct a graph $G$ with $3n$ vertices that correspond to the variables in $F$.
- For any clause in $F$, connect by three edges the corresponding vertices in $G$.
- Connect all pairs of vertices corresponding to a variable $x$ and its negation $\neg x$.
- $F$ is satisfiable if and only if $G$ contains an independent set of size $n$.

## NP-hard problems

- *Decision problem:* solution is either yes or no
- Example: Traveling salesman decision problem:
  Given a network of cities, distances, and a number B, does there exist a tour with length $\leq$ B?
- *Search problem:* find an object with required properties
- Example: Traveling salesman optimization problem:
  Given a network of cities and distances, find a shortest tour.
- Decision problem *NP*-complete $\Rightarrow$ search problem *NP*-hard
- *NP-hard problems:* at least as hard as *NP*-complete problems

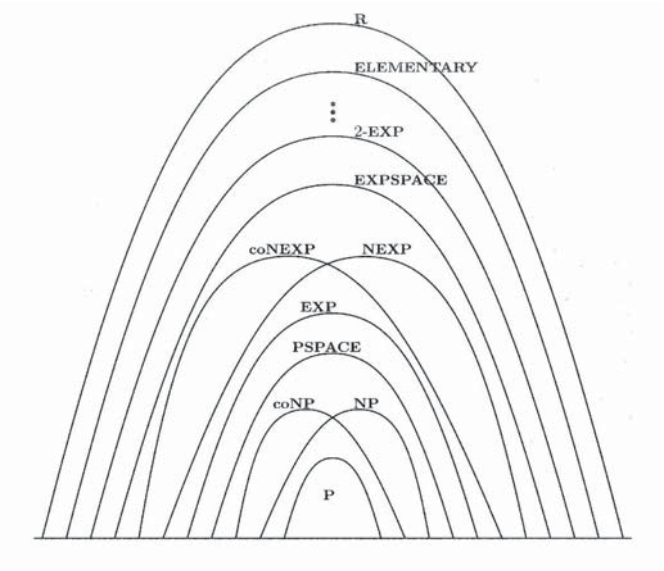
## NP-hard problems in bioinformatics

- Multiple sequence alignment *Wang/Jiang 94*
- Protein folding *Fraenkel 93*
- Protein threading *Lathrop 94*
- Protein design *Pierce/Winfree 02*
- ...

## Further complexity classes

| | |
|---|---|
| *coNP:* | Problems whose complement is in *NP* |
| *PSPACE:* | Problems solvable in polynomial space |
| *EXPTIME:* | Problems solvable in exponential time |
| ⋮ | |

## Hierarchy




## Literature


- J. E. Hopcroft and J. D. Ullman: Introduction to automata theory, languages and computation. Addison-Wesley, 1979
- M. R. Garey and D. S. Johnson: Computers and intractability. A guide to the theory of NP-completeness. Freeman, 1979
- C. H. Papadimitriou: Computational complexity. Addison-Wesley, 1994
- S. Arora and B. Barak: Computational complexity - a modern approach. Camdridge Univ. Press, 2009