ABHYANKAR’S CONJECTURES

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Dedicated to the memory of Michel Raynaud

NOTATION

Let $k$ be an algebraically closed field of characteristic $p > 0$. For every finite group $G$ we will denote by $p(G)$ the subgroup of $G$ generated by its $p$-Sylow subgroups. We will say that a group $G$ is a quasi-$p$-group if $G = p(G)$.

1. Introduction

In 1957 Abhyankar formulated the following conjecture.

Conjecture 1.1 (Abhyankar’s Conjecture for affine curves). Let $X$ be a smooth projective curve of genus $g$ defined over $k$ and $S$ a non-empty set of closed points of $X$ of cardinality $r$. A finite group $G$ is the Galois group of some finite étale Galois cover of $X - S$ if and only if $G/p(G)$ admits a generating set of cardinality at most $2g + r - 1$.

In particular the conjecture says that a finite group $G$ is the Galois group of some Galois cover of $\mathbb{A}^1_k$ if and only $G$ is a quasi-$p$-group.

The conjecture is now completely proven by the work of Serre, Raynaud and Harbater. Serre has proven the conjecture when $X - S = \mathbb{A}^1_k$ and $G$ is assumed to be solvable. Later Raynaud succeeded to prove the entire conjecture over $X - S = \mathbb{A}^1_k$, using the case proven by Serre. Finally Harbater have proven the full conjecture.

The seminar will take inspiration from [BLR00]. We will see a proof of the entire conjecture and we will introduce the tools involved. As in [BLR00], we have preferred Pop’s proof [Pop95] of the general Abhyankar’s conjecture rather than Harbater’s proof.

BACKGROUND

We will take as a background the theory of schemes and formal schemes, the definition of the étale fundamental group and the étale cohomology. We will recall the necessary facts on the ramification theory and the semi-stable reduction of curves, group’s cohomology and rigid geometry (using Tate’s approach).

2. Plan of the seminar

conjectures: one over $\mathbb{F}_p$ and the other on inertia groups. Brief overview on the higher dimensional case. Belyi’s type theorems in positive characteristic.


3. Local-to-global extension of covers. (Marco, 30.04.18), Prove Theorem 1.4.1 in [Kat86]. See also [BLR00, §14]. Present the example in [BLR00, 14.2.5.(b)].

4. Serre’s proof in the solvable case. (Grétar, 07.05.18) Explain the main result of [Ser90].

5. Rigid analytic spaces. (Julian, 14.05.18) Main reference: [Bos14, §2 to §5] Brief introduction to affinoid spaces: Tate algebras, maximum principle, basic properties of affinoid algebras. Affinoid subdomains (definition + Theorem §3.3.20 without proof will be enough). State Tate’s acyclicity theorem. Introduce the strong Grothendieck topology. Give the definition of rigid analytic spaces. Use the ball as a toy example to explain the definitions you give.

6. Coherent modules and rigid analytic GAGA. (Efstathia, 28.05.18) Main reference: [Bos14, §5 to §6]. Coherent modules and their cohomology. Proper mapping theorem (without proof). GAGA theorems (theorems 6.4.11-12-13).

7. Raynaud’s generic fiber. (Yun, 04.06.18) Raynaud’s generic fiber [Bos14, §8] or [BLR00, §1.2]. Runge’s theorem in rigid geometry [Ray94, Corollary 3.5.2] and the other results in [BLR00, §1.3].

8. Formal patching. (Tanya, 11.06.18) Here you can choose among many references. The main thing is to show Theorem 1 in [HS99]. Another place where you can read it is [BLR00, §10.3]. Then you can prove [BLR00, Theorem 10.4.1].

9. Raynaud’s proof I: Construction of coverings via rigid geometry. (18.06.18) Main reference: [Ray94]. State Theorem 2.2.1 and notice that 2.2.1.(1) is the result of Serre we have already proven. Show how Theorem 2.2.3 implies Theorem 2.2.1.(2). Prove Theorem 2.2.3 (§5 in Raynaud) assuming all the propositions in §3 and §4. See also [BLR00, §15.1-2].

10. Raynaud’s proof II: Extension and algebraization. (Yun, 25.06.18) Main reference: [Ray94]. Prove 3.4.1, 3.4.6, 3.4.8, 4.2.6, 4.2.8. See also [BLR00, §15.3-4].

11. Raynaud’s proof III: Last case using semi-stable curves. (Otabe, 02.07.18) Main reference: [Ray94]. Explain the idea of the proof of Theorem 2.2.1.(3). Explain §6.3. See also [BLR00, §16.1].

12. Raynaud’s proof IV: The combinatorial step. (Fei, 09.07.18) Main reference: [Ray94]. Explain §6.4. Prove Theorem 2.2.1.(3) following §6.5. See also [BLR00, §16.2-3].

13. Pop’s proof of the general case. (16.07.18) Prove the general Abhyankar’s conjecture as in [Pop95]. See also [BLR00, §15.5].
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References


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