On Quillen's simplicial interpretation of Hochschild cohomology

by Reiner Hermann

Abstract: The theory of Hochschild (co)homology has, since it was introduced by Gerhard Hochschild in 1945, developed into various areas of mathematics, such as differential geometry, deformation theory, K-theory and representation theory. A main object of study is the Hochschild cohomology ring, which carries rich algebraic structure, such as a graded commutative product and a compatible graded Lie bracket, which can be though of as a (non-commutative) analogue of the Schouten-Nijenhuis-bracket of (poly)vector fields known from Poisson geometry.

The aim of this talk is twofold: First, we discuss the algebraic version of Hochschild cohomology, along with its product structure, and explain how the Lie bracket may be understood from the perspective of homotopy theory due to an interpretation by Stefan Schwede. Then, following ideas of Daniel Quillen, we will give an indication of how one may realise Hochschild cohomology as graded derivations of certain simplicial resolutions. We end by asking whether the obvious Lie algebra structure on the latter object agrees with the one on Hochschild cohomology, when translated appropriately, and try to motivate as to why this might be a question of significance.