Equivariant methods in Geometric Combinatorics - An Overview

Lecture 2

The first lecture was concluded with the question how the topological Tverberg theorem for prime powers can be proved. This is the point where topological tools beyond Borsuk–Ulam and Dold's theorem have to be used.

In this lecture we introduce the notions of group cohomology, equivariant cohomology and Fadell-Husseini ideal valued index theory necessary for the proof of the topological Tverberg theorem for prime powers. The aim of this lecture is the proof of the following theorem.

Theorem 1. *Let p be a prime, $G = (\mathbb{Z}_p)^n$ and X be a compact G-space. Then*

$$X^G \neq \emptyset \quad \Leftrightarrow \quad \pi_X^* : H_X^*(\mathrm{pt}) \to H_G^*(X) \text{ is a monomorphism,}$$

where $\pi_X : X \to \mathrm{pt}$ is a projection and π_X^* an induced map in equivaraint cohomology.