Topology 3

Problem Set 6 H. Reich/F. Lenhardt
SS 2013 Due: 29.05.

Exercise 1
Let $f\colon X \to Y$ be a (unpointed) homotopy equivalence. Show that for all $x_0 \in X$ f induces an isomorphism

$$\pi_n(f)\colon \pi_n(X, x_0) \longrightarrow \pi_n(Y, f(x_0)).$$

Exercise 2
Let $X_0 \hookrightarrow X_1 \hookrightarrow X_2 \hookrightarrow \dots$ be a sequence of closed inclusions of Hausdorff spaces and let $X = \operatorname{colim} X_i$. Let $K \subset X$ be compact. Show:

(i) If you choose for $i \in \mathbb{N}_0$ with $K \cap X_i \neq 0$ a point in $K \cap X_i$, the arising subset of X is finite.

(ii) Each compact subspace $K \subset X$ is contained in one of the X_i.

(iii) For $x_0 \in X_0$, the natural map

$$\operatorname{colim} \pi_n(X_i, x_0) \longrightarrow \pi_n(X, x_0)$$

is an isomorphism.

Exercise 3
Let X be the universal cover of $S^1 \vee S^2$ and $x_0 \in X$ a basepoint. Show that $\pi_2(X, x_0)$ is not finitely generated.

Hint: You may use $\pi_2(S^2) \cong \mathbb{Z}$ and the exercise above.

Exercise 4
Let $p\colon E \to B$ be a Hurewicz fibration. For a path $\alpha\colon I \to B$ from b to b' let $[\tau_{[\alpha]}] \in [p^{-1}(b), p^{-1}(b')]$ be the fiber transport defined in the lecture. Show that

$$\left[\tau_{[\alpha * \beta]}\right] = \left[\tau_{[\alpha]} \circ \tau_{[\beta]}\right].$$