Freie Universität Berlin

Topology 3

Problem Set 5 — H. Reich/F. Lenhardt

SS 2013 — Due: 22.05.

Exercise 1

Let $i\colon A \to B$ and $j\colon B \to C$ be cofibrations. Prove that also $j \circ i$ is a cofibration.

Exercise 2

Let $n \geq 1$. Find a CW-complex X with

$$H_i(X;\mathbb{Z}) = \begin{cases} \mathbb{Z} & i = 0 \\ \mathbb{Q} & i = n \\ 0 & \text{sonst.} \end{cases}$$

Hint: Compute the colimit $\mathbb{Z} \xrightarrow{\cdot 2} \mathbb{Z} \xrightarrow{\cdot 3} \mathbb{Z} \xrightarrow{\cdot 4} \mathbb{Z} \longrightarrow \ldots$

Exercise 3

Let G be a topological group and $X = (X, x_0)$ a pointed space.

(i) Define a group structure on the set $[X, G]$ of pointed homotopy classes of maps.

(ii) Show that the group structure on $[\Sigma X, G]$ defined in the lecture agrees with the one you defined in i).

(iii) Show that $[\Sigma X, G]$ is an abelian group.

Exercise 4

Let $X = (X, x_0)$ be a pointed space and $i\colon X \to \Sigma X$ the inclusion. Prove or disprove:

(i) For all $n \geq 1$, the map $i_*\colon \pi_n(X) \longrightarrow \pi_n(\Sigma X)$ is the zero map.

(ii) For all $n \geq 1$, the map $i_*\colon \pi_n(X) \longrightarrow \pi_n(\Sigma X)$ is an isomorphism.