Topology 3

Problem Set 4 — H. Reich/F. Lenhardt
SS 2013 — Due: 15.5.

Exercise 1
Let $X_i = [0,\infty)$ for all $i \in \mathbb{N}$ and let $f_i\colon X_i \to X_{i+1}$ be given by $f(x) = x-1$ if $x \geq 1$ and $f(x) = 0$ otherwise. Determine $\operatorname{colim} X_i$.

Exercise 2
For $S^1 \subset \mathbb{C}$ and $I = [0,1]$ let $f\colon S^1 \to I$ und $g\colon I \to S^1$ be given by $f(z) = \frac{1}{2}\left(\operatorname{Re}(z)+1\right)$ and $g(t) = e^{\pi i t}$.

Determine $\operatorname{colim}\left(S^1 \to I \to S^1 \to I \to S^1 \to \ldots\right)$.

Exercise 3
Prove or disprove:

(i) For a sequence $M_0 \xrightarrow{f_0} M_1 \xrightarrow{f_1} M_2 \xrightarrow{f_2} \ldots$ of R-linear maps and an R-module N, the natural map

$$\operatorname{colim}_i \left(N \otimes_R M_i\right) \longrightarrow N \otimes_R \operatorname{colim}_i M_i$$

is an isomorphism

(ii) For a sequence $M_0 \xleftarrow{f_0} M_1 \xleftarrow{f_1} M_2 \xleftarrow{f_2} \ldots$ of R-linear maps and an R-modulel N, the natural map

$$N \otimes_R \lim_i M_i \longrightarrow \lim_i \left(N \otimes_R M_i\right)$$

is an isomorphism.

Exercise 4
Let A be a $\mathbb{Z}$-module and p a prime.

(i) Show that multiplication by p induces an isomorphism on

$$\lim\left(\ldots \xrightarrow{\cdot p} A \xrightarrow{\cdot p} A \xrightarrow{\cdot p} A\right)$$

(ii) Compute the above limit for $A = \mathbb{Z}$, $A = \mathbb{Z}/p^n\mathbb{Z}$ and $A = \mathbb{Z}/q^n\mathbb{Z}$, where $n \geq 1$ and q is coprime to p.