Topology 3

Problem Set 3 — H. Reich/F. Lenhardt
SS 2013 — Due: 08.05.

Exercise 1

(i) Compute
$$\dim_{\mathbb{F}_5} H_{15}(S^3 \times \mathbb{R}\mathrm{P}^5 \times \mathbb{H}\mathrm{P}^\infty; \mathbb{F}_5).$$

(ii) Detemine the cup product structure on $H^*(\mathbb{C}\mathrm{P}^\infty/\mathbb{C}\mathrm{P}^1; \mathbb{Z})$.

Exercise 2
Let $\mu^* \colon h^* \to k^*$ be a natural transformation of generalized cohomology theories with values in $\mathbb{Z}$-modules. Prove or disprove:

(i) Is $\mu^*_{\mathrm{pt}} \colon h^*(\mathrm{pt}) \to k^*(\mathrm{pt})$ injective, then for all CW-pairs (X, A) also $\mu^*_{(X,A)} \colon h^*(X, A) \to k^*(X, A)$ is injective.

(ii) Is $\mu^*_{\mathrm{pt}} \colon h^*(\mathrm{pt}) \to k^*(\mathrm{pt})$ surjective, then for all CW-pairs (X, A) also $\mu^*_{(X,A)} \colon h^*(X, A) \to k^*(X, A)$ is surjective.

Exercise 3
For CW-complexes X with $\dim_{\mathbb{Q}} H_i(X; \mathbb{Q}) < \infty$ for all $i \geq 0$ we define a formal power series via
$$p_X(t) = \sum_{i=0}^{\infty} \dim_{\mathbb{Q}} H_i(X; \mathbb{Q})\, t^i\,.$$
Find a formula for $p_{X\times Y}(t)$ in terms of $p_X(t)$ and $p_Y(t)$.

Exercise 4
Let X be a topological space. A subset $A \subset X$ is called *f-closed* if the following holds: If a_n, $n \in \mathbb{N}$ is a sequence with $a_n \in A$ which converges in X to a point $x \in X$, then $x \in A$. A topological space is called *f-space* if each f-closed subset is closed. Show:

(i) Each closed set is f-closed.

(ii) Each k-closed set is f-closed.

(iii) f-spaces are compactly generated.

Hint: $K = \{\frac{1}{n} \mid n \in \mathbb{N}\} \cup \{0\}$ equipped with the subspace topology of $\mathbb{R}$ is compact.